ARTICLE

Three Tests for Practical Evaluation of Partisan Gerrymandering

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Abstract. Since the U.S. Supreme Court's *Davis v. Bandemer* ruling in 1986, partisan gerrymandering for statewide electoral advantage has been held to be justiciable. The existing Supreme Court standard, culminating in *Vieth v. Jubelirer* and *LULAC v. Perry*, holds that a test for gerrymandering should demonstrate both intents and effects and that partisan gerrymandering may be recognizable by its asymmetry: for a given distribution of popular votes, if the parties switch places in popular vote, the numbers of seats will change in an unequal fashion. However, the asymmetry standard is only a broad statement of principle, and no analytical method for assessing asymmetry has yet been held by the Supreme Court to be manageable. This Article proposes three statistical tests to reliably assess asymmetry in state-level districting schemes: (1) an unrepresentative distortion in the number of seats won based on expectations from nationwide district characteristics; (2) a discrepancy in winning vote margins between the two parties; and (3) the construction of reliable wins for the party in charge of redistricting, as measured by either the difference between mean and median vote share, or an unusually even distribution of votes across districts. The first test relies on computer simulation to estimate appropriate levels of representation for a given level of popular vote and provides a way to measure the effects of a gerrymander. The second and third tests, which can be used to help evaluate redistricting intent, rely on well-established statistical principles and can be carried out using a hand calculator without examination of maps or redistricting procedures. I apply these standards to a variety of districting schemes, starting from the original "Gerry-mander" of 1812, up to modern cases. In post-2010 congressional elections, partisan gerrymandering in a handful of states generated effects that are larger than the total nationwide effect of population clustering. By applying these standards in two recent cases, I show that Arizona legislative districts (*Harris v. Arizona Independent Redistricting Commission*).

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Commission) fail to qualify as a partisan gerrymander, but Maryland’s congressional districts (Shapiro v. McManus) do. I propose that an intents-and-effects standard based on these tests is robust enough to mitigate the need to demonstrate predominant partisan intent. The three statistical standards offered here add to the judge’s toolkit for rapidly and rigorously identifying the partisan consequences of redistricting.
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Introduction

Partisan gerrymandering, in which geographical jurisdictions are divided to give special advantage to one political group over others, is quite old, dating to the establishment of Pennsylvania’s assembly districts in 1705. The term “Gerry-mander” was later coined in 1812 to mock an oddly shaped district encompassing northern parts of Essex County, Massachusetts. The broader target of editorial scorn, however, was the overall goal of gaining more seats at the statewide level than the party’s support among the population would normally justify. For the “Gerry-mander,” redistricters from Massachusetts—specifically, Governor Elbridge Gerry’s Democratic-Republican Party—sought to take popular support that was closely divided between their party and the other major party, the Federalists, and divide it among districts to favor their own side. The stratagem worked: Federalists won the two-party vote share by a margin of 51%-to-49% over the Democratic-Republicans, but ended up severely outnumbered in the General Court, with only eleven seats to the Democratic-Republicans’ twenty-nine seats. Federalist voters were packed so that Federalist candidates won an average of 71%-to-29% of the two-party vote in the districts they carried. Democratic-Republicans were distributed to allow wins in a larger number of districts, averaging 56%-to-44% per district. This result exemplifies a central principle of partisan gerrymandering: concentrate voters on a district-by-district basis such that both sides’ wins are reliable, but the redistricting party’s victory margins are smaller than those of the opposing party and are thereby used more efficiently.

The seat advantage gained in a partisan gerrymander represents a distortion arising from the districting process that causes election results to deviate from natural patterns. Such distortions, however, do not necessarily persist over time. In the case of the original “Gerry-mander,” the next election, in 1813, showed a rapid reversal of fortune for the Democratic-Republicans. Public anger over the War of 1812 and over gerrymandering itself led to

3. See GRIFFITH, supra note 1, at 62.
4. Id. at 72-73; A New Nation Votes: American Election Returns 1787-1825, TUFTS DIGITAL COLLECTIONS & ARCHIVES (June 24, 2009), http://elections.lib.tufts.edu [hereinafter Lampi Collection].
5. Lampi Collection, supra note 4.
6. Id.
7. Id.
increased Federalist turnout, and a 56%-to-44% popular-vote victory, with an outcome of twenty-nine Senate seats to the Democratic-Republicans’ eleven.\(^9\) This perfect reversal of outcomes was achieved with only a five percent increase in the Federalists’ vote share. Such a dramatic swing was possible because Democratic-Republican-leaning districts were engineered to deliver extremely narrow victories, so that a small swing in opinion was sufficient to influence many races.

The example of Massachusetts in 1812 and 1813 shows that a partisan gerrymander’s effects can be reversed if voter sentiments change sufficiently. A gerrymander can also weaken if voters physically change residence. When district boundaries are carefully constructed based on the pattern of voter residence at a single point in time, it is more likely than not that voter mobility will tend to dissipate the advantage, much as a child’s carefully built sandcastle, once left unattended, will erode with the wind.

Finer-grained drawing of boundaries and technological advances have since opened the possibility of drawing more sophisticated gerrymanders that potentially lead to more secure and lasting advantages for the party in charge of redistricting. Several factors come into play.

First, redistricting was once done on a county-by-county basis.\(^10\) Detailed census and voter-registration information is now available, allowing redistricters to construct districts on a block-by-block basis.\(^11\) Districting software, in both commercial and freely available varieties, allows users to access this information to explore many scenarios in rapid succession and to create boundaries that separate different populations of voters in exquisite details. Professionals use proprietary software to draw districts, but even activists and ordinary citizens can enter the fray using free software such as Dave's Redistricting App.\(^12\)

Second, voters themselves have tended to cluster into Democratic- and Republican-prefering communities. Generally speaking, Democratic voters are found more often in regions of higher population density, and Republican voters in regions of lower population density. These tendencies have

\(^9\) Lampi Collection, supra note 4.
intensified in recent years as part of a phenomenon termed “the Big Sort.”
This sorting leads voters to become self-aggregated into easy-to-handle contiguous chunks, within which partisan preference is strong in one direction or the other. Overall, reliable partisan voting and the Big Sort create geographic patterns that make it easier to gerrymander. In this way, polarization can facilitate gerrymandering. Furthermore, safely held seats, whether they arise from polarized communities or from gerrymandering, insulate representatives from voter preference.

Based on analysis in the 1990s, the effects of partisan congressional gerrymanders have been estimated to last for multiple election cycles, but with the potential to diminish after even one election cycle. The Big Sort may allow redistricting to have longer-lasting effects as neighborhood-level partisan tendencies become more stable. In addition, changes in technical tools and population clustering, as well as a greater awareness of the advantages of aggressive districting, further enhance the possibility that gerrymandered districts may be more durable now than they were even ten years ago.

Often, a two-party system exhibits a high degree of partisan symmetry: if the major parties were to switch vote share, they would also come close to switching their share of seats in the legislative body. However, partisan gerrymandering has reached recent extremes of asymmetry as an increasing number of state governments have come under one-party rule. All these
factors working together—the Big Sort, more detailed data, computer-based
districting, and single-party rule—provide easier routes to give undue
advantage to whichever political party controls redistricting. These factors
magnify the need for a manageable standard to define—and potentially curb—
partisan gerrymandering.

In this Article, I present three tests that address the problem of detecting
extreme deviations from partisan symmetry. First, in Part I, I review court
precedents that establish the desirability of partisan symmetry as an outcome, a
concept that can be used to help define a partisan gerrymander. In Part II, I
describe mathematical approaches, grounded in longstanding statistical
practice, to detect partisan asymmetry. I present two analyses: one that
measures effect, which I define as the number of seats that are gained by a
gerrymander, and one that detects intent, which I define as a pattern of
district-level partisan outcomes that is unlikely to have arisen by chance. The
number-of-seats measure specifically overcomes the central difficulty that
representation is not necessarily proportional to public support. Nonproportionality has long been known to arise naturally from the winner-
take-all nature of individual elections.18 My calculation of effects replaces the
intuitive, but incorrect, ideal of proportionality.

In Part III, I use these analyses to construct three tests to evaluate cases of
gerrymandering. I apply my tests to example cases, starting with the original
Gerry-mander of 1812, up to post-2010 congressional districting plans in all
fifty states. Further, I also consider two recent cases that have come before the
Supreme Court: the Maryland congressional delegation, in Shapiro v.
McManus,19 and the Arizona state legislative districts, in Harris v. Arizona
Independent Redistricting Commission.20 The results of the tests support the idea
that gerrymandering has distorted the composition of Maryland’s
congressional delegation and has made it unresponsive to changes in voter

AM. POL. SCI. REV. 540, 542-43 (1973). For example, in a two-party system, it is
theoretically possible for one political party to win forty-nine percent of the vote in
every district, yet not win a single delegate. Although such an extreme case is highly
improbable, strong deviations from proportionality are nevertheless an inherent risk
of a winner-take-all district system. From a democratic standpoint, a central question
is how to avoid the most extreme distortions. Actual nongerrymandered outcomes are
considerably less distorted than the extreme hypothetical scenario described above.

sentiment. By contrast, Arizona legislative districts do not show significant asymmetry. In Part IV, I conclude by suggesting ways in which these tests can be used to construct a manageable standard for use by courts and legislatures. These tests are available for online use at http://gerrymander.princeton.edu.

I. Background

A. Current Legal Constraints on How a Partisan Gerrymander May Be Defined

The U.S. system of representative democracy contains at its core a tension based on the fact that all federal, and many state and local, legislators are elected in single-member districts. In such a system, citizens are assigned to districts where they elect one legislator. A cardinal advantage of this system is that a specific legislator in the House of Representatives or a state legislative chamber represents every citizen. It is a common trope to speak of contacting one's representative to seek redress of government-related issues, and this system provides citizens with a direct path for doing so.

Interposed in this seemingly straightforward path between citizens and legislators is the process by which districts are drawn. District maps are redrawn anew following each decade's census, which determines the distribution of representatives in the House of Representatives among the states. Given its number of representatives, each state has responsibility for drafting U.S. House and state legislative districts, a process that is constrained by natural variations in population, laws that govern the drawing of boundaries, compromises made during the legislative process, and whether voting laws applied by the Justice Department and courts allow a particular set

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22. Similar problems exist at the level of state legislatures. The analysis described in this Article is also applicable to evaluating state-level districting results.


of boundaries to withstand evaluation under the Voting Rights Act.\textsuperscript{25} Virtually all districting schemes resulting from this process generate representation that is not directly proportional to public support, a well-known consequence of the winner-take-all nature of individual elections.\textsuperscript{26} Despite this difficulty and the baroque, almost rococo nature of the districting process, at a national level the party that receives more votes usually receives the majority of seats.\textsuperscript{27}

When litigants challenge districting plans for partisan gerrymandering, they assert that voters have lost the ability to elect representatives that fairly reflect their views. Also, redistricting efforts are said to confer specific advantage to one political party at the expense of another. In most partisan gerrymanders, the districting scheme results in the election of delegations that do not naturally reflect the overall preferences of the state's voters. Two fundamental strategies for achieving this outcome are “packing,” in which a district is heavily loaded with supporters of the opposing party so that their votes are wasted, and “cracking,” in which a bloc of votes is split across districts to dilute their impact and prevent them from contributing to a majority in any one district.\textsuperscript{28}

An important component of remedying a gerrymandering offense is identifying who is harmed and how. The most obvious harm from partisan gerrymandering is representational. Partisan gerrymandering creates a situation in which the same overall statewide vote share would lead to a very different level of representation for the redistricting party and its opposing target: “For example, in the Pennsylvania congressional election of 2012, Democrats won only 5 out of 18 congressional House seats, despite winning slightly more than half of the statewide vote.”\textsuperscript{29} Democratic winners were packed into districts where they won an average of 76% of the vote, while


\footnotesize{\textsuperscript{26} Tufte, supra note 18, at 542-43.}

\footnotesize{\textsuperscript{27} Sam Wang, Opinion, The Great Gerrymander of 2012, N.Y. TIMES (Feb. 2, 2013), http://nyti.ms/WMCC7Q.}

\footnotesize{\textsuperscript{28} LEVITT, supra note 11, at 57-58.}

\footnotesize{\textsuperscript{29} Sam Wang, Opinion, Let Math Save Our Democracy, N.Y. TIMES (Dec. 5, 2015), http://nyti.ms/1YR7AdU.}
Republican winners won an average of 59%.\textsuperscript{30} In this way, the artful drawing of district boundaries can create representational asymmetry between the two major political parties.

Partisan gerrymandering can also chill a voter's freedom to choose between political parties. In gerrymandered districts, the noncompetitive nature of the general election leaves the primary election as the only avenue for voters to affect their representation. Such a situation creates a powerful incentive to compel voters to join the dominant political party, even if that party's issue positions do not encompass his or her political views. Since a partisan gerrymander creates noncompetitive districts for both major parties, voters on both sides may potentially feel the chill.

The use of redistricting for partisan advantage has taken on new importance in a polarized political environment, and nonpartisan congressional scholars have identified gerrymandering as a substantial risk to representative democracy.\textsuperscript{31} Voters, however, are not without recourse. Partisan gerrymandering has formed the basis of many recent judicial challenges to redistricting, including multiple challenges since the 2010 census.\textsuperscript{32}

The justiciability (at least in principle) of partisan gerrymandering arises from a series of Supreme Court cases starting with \textit{Davis v. Bandemer},\textsuperscript{33} and continuing with \textit{Vieth v. Jubelirer}\textsuperscript{34} and \textit{League of United Latin American Citizens (LULAC) v. Perry}.\textsuperscript{35} In \textit{Davis v. Bandemer}, in response to Indiana Democrats' assertion that they were systematically disadvantaged by the state's legislative map, the Supreme Court held that partisan gerrymandering claims are justiciable.\textsuperscript{36} Although the Court did not rule in the litigants' favor, it did lay out a cause of action based on a two-prong test: (1) intent—an established purpose to create a legislative districting map to disempower the voters of one

\begin{thebibliography}{99}
\bibitem{Litigation} See \textit{Litigation in the 2010 Cycle, All About Redistricting}, http://redistricting.lls.edu/cases.php#sct (last visited June 6, 2016) (listing redistricting challenges pending before the Supreme Court).
\bibitem{478 U.S. at 125.} 478 U.S. at 125.
\end{thebibliography}
party; and (2) effect—proof that an election based on the contested districting
scheme led to a distorted outcome.37

Partisan gerrymandering’s unconstitutionality rests first on the potential
rationale of the Fourteenth Amendment’s Equal Protection Clause.38 An equal
protection rationale might suggest the possibility of taking a disparate impact
approach to partisan gerrymandering. In the housing discrimination case

Village of Arlington Heights v. Metropolitan Housing Development Corp., the Court
established a framework in which courts evaluate a number of factors to
identify housing discrimination in part by considering disparate impact and/or
disparate treatment of groups of differing socioeconomic or racial
characteristics.39 As I describe in Part I.C below, however, the Supreme Court
has thus far not adopted standards resembling the Arlington Heights criteria in
the context of partisan gerrymandering.

Indeed, as seen in Vieth, the Court has developed an explicit distinction
between racial and partisan gerrymandering. The question presented in Vieth
was whether Pennsylvania’s congressional districts constituted a partisan
gerrymander.40 Five Justices voted to dismiss the claim.41 Justice Scalia wrote a
plurality opinion joined by Chief Justice Rehnquist, Justice O’Connor, and
Justice Thomas.42 Justice Scalia wrote, “[T]o the extent that our racial
gerrymandering cases represent a model of discernible and manageable
standards, they provide no comfort” in the partisan gerrymandering context.43
By contrast, in his dissent, Justice Stevens explained that he “would apply the
standard set forth in the Shaw [racial gerrymandering] cases” in “evaluating a
challenge to a specific district” on partisanship grounds.44

In addition to a Fourteenth Amendment rationale, Justice Kennedy
suggested a basis for determining partisan gerrymandering under the First
Amendment’s protection of speech and association.45 Unlike ethnicity or
socioeconomic status, identification with a political party can be changed with
little effort. In this respect, partisan identification can be regarded as an act of

37. Id. at 127 (plurality opinion) (agreeing with the district court that plaintiffs must prove
both discriminatory intent and effect).
38. Id. at 122-23 (majority opinion); Vieth, 541 U.S. at 314 (Kennedy, J., concurring in the
judgment) (“[T]hese allegations involve the First Amendment interest of not
burdening or penalizing citizens because of their participation in the electoral
process, . . . their association with a political party, or their expression of political
views.”).
40. 541 U.S. at 271 (plurality opinion).
41. Id. at 270.
42. Id.
43. Id. at 286.
44. Id. at 339 (Stevens, J., dissenting).
45. Id. at 314 (Kennedy, J., concurring in the judgment).
speech or free association. Justice Kennedy described the “First Amendment interest of not burdening or penalizing citizens because of their participation in the electoral process, their voting history, their association with a political party, or their expression of political views. Under general First Amendment principles, those burdens in other contexts are unconstitutional absent a compelling government interest.”46

Although Justice Kennedy left this door open, he did not articulate an actual standard to evaluate partisan gerrymandering claims under the First Amendment. The harms I delineate above suggest two possibilities. First, packing voters into districts based on their partisan affiliation may constitute an infringement of the right to public self-expression, or freedom of speech. Second, chilling of partisan choice may constitute an infringement of freedom of association. Together, these harms constitute a form of viewpoint discrimination. Thus, the purposeful creation of lopsided districts can potentially be linked to First Amendment principles.

Since the Court’s holding in Bandemer that partisan gerrymandering claims are justiciable, the Court has struggled to identify a manageable standard, i.e., one that provides a reliable and usable determination of whether an offense has occurred. In Bandemer, the Justices described the effects prong in general terms. Advocating for an analysis of an entire districting plan, Justice White explained that “[s]tatewide, . . . the inquiry centers on the voters’ direct or indirect influence on the elections of the state legislature as a whole”47 and acknowledged that this was “of necessity a difficult inquiry.”48 But eighteen years later in Vieth, the plurality opinion stated that no acceptable standard had been established in the intervening time, and therefore it was time to abandon the search.49 The Court in Vieth was notably divided, culminating in five separate opinions.50 In his separate concurrence, Justice Kennedy provided the fifth vote against invalidating the districts in Pennsylvania but left the door open for a remedy in future cases if a clear standard could be established.51 The dissenting four Justices voted in favor of a finding of partisan gerrymandering and offered several possible standards, but none was backed by a majority of

46. Id.
48. Id. at 143.
49. 541 U.S. at 279, 281 (plurality opinion).
50. See id. at 271; id. at 306 (Kennedy, J., concurring in the judgment); id. at 317 (Stevens, J., dissenting); id. at 343 (Souter, J., dissenting); id. at 355 (Breyer, J., dissenting).
51. Id. at 306 (Kennedy, J., concurring in the judgment) (“I would not foreclose all possibility of judicial relief if some limited and precise rationale were found . . . .”).
Justices.52 This judicial stalemate was left unaltered by LULAC, a case on mid-decade redistricting in Texas.53

In this Article, I present three tests that address concerns expressed in Justice Scalia’s and Justice Kennedy’s opinions in Vieth. The tests are rooted in both statistics and a principle of symmetry that has attracted favorable comment from six Justices across multiple opinions in LULAC.54 Although the Justices were not precise in their approbation, the symmetry concept did appear to capture their intuitions better than any other effort to date. In this Article, I expand on this intuitive impulse by adding mathematical rigor previously absent from the Court’s partisan gerrymandering jurisprudence. I present tests that provide a judicially manageable standard—based in the Constitution—for identifying partisan asymmetry. As the Court changes with the passing of Justice Scalia, my approach provides a highly natural set of tests that may appeal to Justices who are willing to find partisan gerrymandering justiciable.

B. Searching for a Manageable Standard: The Current State of Play

The Court has repeatedly expressed the desire to find a manageable standard for partisan gerrymandering. In Vieth, Justice Kennedy explained:

When presented with a claim of injury from partisan gerrymandering, courts confront two obstacles. First is the lack of comprehensive and neutral principles for drawing electoral boundaries. No substantive definition of fairness in districting seems to command general assent. Second is the absence of rules to limit and confine judicial intervention.55

These concerns are longstanding. In Bandemer, Justice O’Connor expressed concern that the plurality’s standard “will over time either prove

52. See supra note 50 and accompanying text.
54. See id. at 420 (presuming that a partisan gerrymandering challenge could be litigated based on actual election results but not “in a hypothetical state of affairs,” and stating that “[w]ithout altogether discounting its utility in redistricting planning and litigation, I would conclude asymmetry alone is not a reliable measure of unconstitutional partisanship”); id. at 473 n.11 (Stevens, J., concurring in part, dissenting in part) (describing asymmetry as one of eight criteria for determining effects-based violations); id. at 468 n.9 (describing the symmetry standard as “a helpful (though certainly not talismanic) tool”); id. at 466 (finding that the challenged plan was “inconsistent with the symmetry standard” and asserting that the “symmetry standard . . . is undoubtedly ‘a reliable standard’ for measuring a burden . . . on the complainants’ representative rights” (quoting id. at 418 (majority opinion))); id. at 483 (Souter, J., concurring in part, dissenting in part) (explaining that he “do[es] not rule out the utility of a criterion of symmetry” as the Court’s “interest in exploring this notion is evident”). For a further review of these statements in LULAC, see Bernard Grofman & Gary King, The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering After LULAC v. Perry, 6 ELECTION L.J. 2, 4-5 (2007).
55. 541 U.S. 267, 306-07 (Kennedy, J., concurring in the judgment).
unmanageable and arbitrary or else evolve towards some loose form of proportionality." Justice Scalia quoted Justice O'Connor in his plurality opinion in *Vieth*, expressing pessimism that such standards could ever be established. The *LULAC* opinion, however, suggested partisan symmetry as a fresh start. Inspired by *LULAC*, this Article builds upon partisan symmetry to develop statistical ideas that are aimed at overcoming or bypassing prior concerns.

Considering the multiple foregoing criticisms, it is worth reviewing some previous proposed criteria for partisan gerrymandering that were offered in *Vieth* and *LULAC*, but which the Court rejected or did not embrace. Upon closer examination, those decisions point toward criteria for what an acceptable test might look like.

1) Majority of votes, majority of seats. In *Vieth*, the second part of appellants' proposed effects standard suggested that the "totality of circumstances' confirms that the map can thwart the plaintiffs' ability to translate a majority of votes into a majority of seats." In his dissent, Justice Breyer described the effect of partisan gerrymandering as the "unjustified use of political factors to entrench a minority in power." Conceptually, the conversion of a majority of votes to a minority of seats is a precursor of the partisan-symmetry concept.

A "majority-majority" standard, however, is vulnerable to variation and chance. As Justice Scalia explained in *Vieth*, "In any winner-take-all district system, there can be no guarantee, no matter how the district lines are drawn, that a majority of party votes statewide will produce a majority of seats for that party." To put these concerns into quantitative terms: the majority-majority standard does not take into account the possibility that an outcome could arise not via skullduggery but by more innocent variations in voting patterns or district-drawing. Although Justice Scalia's hypothetical concern is literally true, it neglects the possibility that a mathematical analysis can offer clarification.

The majority-majority standard could be improved by identifying a "zone of chance"—a range of naturally likely election outcomes in which Justice Scalia's objection might plausibly apply. Under a partisan gerrymandering claim, if the outcome falls outside the zone of chance, Justice Scalia's objection does not apply. Indeed, with statistical methods, it is possible to identify a zone of chance not just in the case of a simple majority but for any given popular-vote outcome. I use statistical analysis to identify zones of chance, which I

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57. 541 U.S. at 282 (plurality opinion).
59. *Vieth*, 541 U.S. at 360 (Breyer, J., dissenting).
60. Id. at 289 (plurality opinion).
define in Part II.A.2 below as ranges of outcome that could have arisen without overall planning from variations in how districts are drawn.61

2) Characteristics of individual districts. Justice Scalia wrote that because all map drawing is inherently political, “[t]he central problem is determining when political gerrymandering has gone too far.”62 Justice Souter suggested that examining individual districts could identify partisan gerrymandering.63 Partisan gerrymandering, however, arises not from single districts but instead emerges from patterns of multiple districts combined. Indeed, a given set of boundaries for any given district might or might not lead to an overall partisan advantage, depending on how the other districts are drawn.

Legislators have long sought to protect individual incumbents and to maximize the advantage for their party. But what is good for an individual incumbent is not always good for his or her party at the statewide level, and vice versa. It is essential to distinguish a single-district gerrymander, which eliminates competition in only one district, from statewide gerrymandering, which consists of an artful pattern of many single-district gerrymanders to distort the overall outcome.64

In single-district gerrymandering, the core technique is to draw a single district’s boundaries circuitously, choosing precise but potentially meandering shapes that increase one candidate’s number of supporters. However, self-interest does not necessarily lead to a statewide antiproportional outcome. As an example, imagine a situation in which incumbents of both parties agree to

61. The zone-of-chance concept is a way to express the concept of significance testing in statistics. Statisticians calculate how far a measurement, such as the number of seats won by a party in a given election, is likely to stray from the expected average. The yardstick for the amount strayed is the “standard deviation,” a quantity denoted by the Greek letter sigma (σ). In this Article, I define the zone of chance as a region within which chance outcomes would fall ninety-five percent of the time and outside the region five percent of the time. Statistics texts refer to this as a “p<0.05” or “α<0.05” standard. The size of the zone of chance is some multiple of sigma, which can be calculated, and is always at least 1.6 times sigma for a bell-shaped curve and 1.75 times sigma for a t-distribution. See also Wang, supra note 29. See generally RICHARD LOWRY, Tests of Statistical Significance: Three Overarching Concepts, in CONCEPTS AND APPLICATIONS OF INFERENTIAL STATISTICS (2000), http://vassarstats.net/textbook/ch7pt1.html (providing an introduction to the concept of significance testing using confidence intervals).

62. Vieth, 541 U.S. at 296 (plurality opinion).
63. Id. at 353 (Souter, J., dissenting).
64. As an example, racial gerrymandering is judged one district at a time, not on a statewide basis. See, e.g., Ala. Legislative Black Caucus v. Alabama, 135 S. Ct. 1257, 1265 (2015); Miller v. Johnson, 515 U.S. 900, 916 (1995); Shaw v. Reno, 509 U.S. 630, 649 (1993). Even more broadly, the word “gerrymander” is colloquially used to describe a range of partisan offenses, including polarization of voters. Such overbroad usage dates back at least a hundred years. See GRIFFITH, supra note 1, at 26-27. In this Article, the term is restricted to the narrower sense of using district boundaries to obtain an advantage for a candidate, faction, or party.
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draw all districts to have a similar advantage, resulting in districts that split 60%-40% in either direction. In such a circumstance, the party with greater popular support must necessarily win more seats.\(^{65}\) Although such incumbent protection is a self-serving act by legislators, it is constitutionally accepted,\(^{66}\) and when it happens symmetrically, it still accurately represents partisan interests. Therefore, it is for good reason that the Vieth decision ruled out the presence of circuitous boundaries as an indicator of partisan gerrymandering.

Circuitous boundaries can also be drawn for nonpartisan reasons, for instance to unify communities of interest or to create districts of near-identical population. Districts may be drawn to contain a large number of minority-group voters, the “majority-minority” districts required under section 2 of the Voting Rights Act.\(^{67}\) These various criteria may have contributed to the rise in circuitousness of boundaries since the 1960s.\(^{68}\) Conversely, relatively straight boundaries do not guarantee a majoritarian outcome. For example, in Michigan, where many congressional district boundaries follow straight north-south and east-west lines for miles at a time, the House popular vote was 53.2% Democratic, 46.8% Republican in 2012, and 50.9% Democratic, 49.1% Republican in 2014, in both cases leading to the election of five Democrats and nine Republicans.\(^{69}\)

In summary, boundaries can serve as an indicator of partisan problems in districting but cannot be used as a sole criterion. I therefore eschew examination of district shapes in constructing my statistical tests.

3) Consideration of districting procedures. As Vieth explained, Justice Powell’s opinion in Davis v. Bandemer proposed to identify “whether district boundaries had been drawn solely for partisan ends to the exclusion of ‘all other neutral

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\(^{65}\) Mathematically, this can be stated as follows. If party \(A\) gets fraction \(V\) of the total two-party vote, and all districts on both sides will be split 60-40, then \(F\), the fraction of \(A\)-favoring districts, must satisfy \(0.6F + 0.4(1-F) = V\). Furthermore, if \(V > 0.5\), i.e., party \(A\) wins the popular vote, then \(F > 0.5\), i.e., the number of \(A\)-favoring districts must also be a majority. This principle is generally true, and is limited only by the fact that for a finite number of districts, the margins of the individual districts would not be precisely 60-40.

\(^{66}\) Vieth, 541 U.S. at 298 (plurality opinion); see Gaffney v. Cummings, 412 U.S. 735, 752-54 (1973).


factors relevant to the fairness of redistricting.” This wording by Justice Powell suggests that it might be possible to detect gerrymandering by comparing the procedures used with more neutral procedures, drawing hypothetical districts, and comparing the predicted hypothetical outcomes with actual election results.

The plurality in Vieth, however, criticized the examination of procedures as being excessively vague. Examination of procedures presents a judge with the question whether a hypothetical alternative plan was drawn with partisan intent. But whenever a district map is drawn, decisions must inevitably be made about whether, and how, to join or split communities. Districting seeks to pursue many goals, including “contiguity of districts, compactness of districts, observance of the lines of political subdivision, protection of incumbents of all parties, cohesion of natural racial and ethnic neighborhoods, compliance with requirements of the Voting Rights Act of 1965 regarding racial distribution, etc.” In addition to these goals, which advance various public interests, legislators and political parties also serve their own interests. Doubtless, the complexity of such a process leads to the “difficult inquiry” cited by Justice White.

One possibility would be to ask what a set of neutral principles might possibly yield. Districting schemes are often tested by detailed procedures such as the JudgeIt algorithm, which has been used by its inventors and other researchers to analyze individual districts. More recently, political scientists Jowei Chen and Jonathan Rodden have developed a sophisticated, automated procedure in which a computer program draws district maps “in a random, partisan-blind manner, using only the traditional districting criteria of equal apportionment and geographic contiguity and compactness of single-member legislative districts.” However, their computerized procedure explicitly omits

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71. See id. at 291.
72. Id. at 284.
73. See Bandemer, 478 U.S. at 143 (plurality opinion).
concerns that might emerge during the legislative process. For example, why, in a densely populated area, should a boundary be as straight as it is in a sparsely populated area? I choose to describe this automated procedure not as a negative criticism, but simply to point out that consideration of districting procedures leads to a proliferation of choices and value judgments—in short, political questions. Drawing districts at random identifies a vast range of possibilities, but does not identify the desirability of a specific outcome.

As an alternative to simulations of the districting process, I suggest that it might be better to use real election results and not use maps at all. Election results nationwide contain a rich source of the consequences of actual legislative dealmaking. In my approach for establishing a manageable standard, I assume that national House districts constitute a sample that reflects accepted standards of districting practice, following a wide variety of geographic, demographic, political, and legal constraints to produce districts of varying partisan composition. In other words, the great give and take of the legislative process in all fifty states provides a desirable setting in which a wide range of prevalent districting standards, measured in terms of outcomes, has been established. For this reason, I use nationwide election results as a baseline for the Analysis of Effects.

4) Predicting partisan loyalties using minor statewide races. Because voters often vote according to their partisan loyalties, it has been suggested that overall voter sentiment can be gauged by examining low-profile statewide races, such as Secretary of State or Attorney General, where candidate-specific factors are ostensibly minimized. However, the Vieth plurality stated that this standard is not judicially manageable. The Vieth Court further noted that “in the 2000 Pennsylvania statewide elections some Republicans won and some Democrats won,” and so these races provided ambiguous guidance as to overall statewide partisan preference. For analyzing congressional or legislative districts, the results of congressional or legislative elections themselves have the advantage of being a direct measure of voter preference for the type of office under dispute and therefore may be a better source of guidance about partisan intention. Given the skepticism surrounding the use of information from other races, the use of results of district-level elections themselves may be more suitable for use in designing a manageable standard.
5) **Partisan symmetry.** As a guiding principle to defining fairness in districting, political scientists Bernard Grofman and Gary King have suggested partisan symmetry:\(^80\): the idea that if the popular vote were reversed, the seat outcome should also reverse. Although a majority of the Court in *LULAC* was favorable to the symmetry concept, a consensus has not yet emerged on how to turn this idea into a specific standard.\(^81\)

The foregoing suggested approaches and criticisms could be viewed with pessimism. In the words of the *Vieth* plurality, the application of the *Bandemer* standard “has almost invariably produced the same result (except for the incurring of attorney’s fees) as would have obtained if the question were nonjusticiable: judicial intervention has been refused.”\(^82\) The *Vieth* plurality further stated that “no judicially discernible and manageable standards for adjudicating political gerrymandering claims have emerged. Lacking them, we must conclude that political gerrymandering claims are nonjusticiable and that *Bandemer* was wrongly decided.”\(^83\) However, the *Vieth* Court did not overturn *Bandemer* because Justice Kennedy’s concurring opinion, the fifth vote against invalidating the districts in Pennsylvania, declined to do so.\(^84\) Still, unless a manageable standard is found, partisan gerrymandering will be nonjusticiable in practice, leaving the *Bandemer* standard toothless.

A more optimistic view is to ask whether the partisan-symmetry idea cited in *LULAC* points to a way forward. An effective and manageable standard should be immune to the criticisms identified above. I suggest that such a standard should have the following minimum qualities: (1) it should be based on the general concept of partisan symmetry;\(^85\) (2) it should not use circuitousness of geographic boundaries or districting procedures; (3) it should not use election results for offices other than the ones that are in dispute; and finally, (4) it should be able to be clearly stated without case-specific or mathematics-intensive assumptions, which might even allow a court to instruct experts on how and where to apply more detailed mathematical or other analysis.

C. **Mathematical Methods Can Identify National- and State-Level Imbalances**

In nationwide elections, majoritarian representativeness is the norm. In the U.S. House of Representatives, when a major party gets more than fifty

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\(^{80}\) See generally Grofman & King, supra note 54, at 4 (explaining partisan symmetry concept).

\(^{81}\) See supra note 54 and accompanying text.

\(^{82}\) 541 U.S. at 279 (plurality opinion).

\(^{83}\) Id. at 281.

\(^{84}\) See supra note 51 and accompanying text.

\(^{85}\) See Grofman & King, supra note 54, at 4.
percent of the vote, it almost always gets over fifty percent of the seats. In thirty-five elections, this basic principle has been violated only twice: in 1996 and in 2012.\textsuperscript{86} National election results, however, give only an aggregated view, and therefore may conceal many sins. Detecting problems in districting requires examination at a state-by-state level.

As previously discussed, antimajoritarian outcomes do not by themselves constitute proof of deliberate distortion of electoral processes. But they do present a preliminary clue that those who draw the districts can influence the relationship between voting and representative outcomes. For example, in the congressional election of 2012, in five states the statewide popular vote favored the opposite party as the delegation that their votes elected: Arizona, Michigan, North Carolina, Pennsylvania, and Wisconsin.\textsuperscript{87} Four of these five antimajoritarian outcomes were enabled by their beneficiary, the Republican Party, which controlled the redistricting process.\textsuperscript{88} Thus antimajoritarian outcomes often, but not always, reflect the partisan interests of those who draw the boundaries. As political parties become a greater predictor of legislative voting patterns, representing partisan loyalties accurately becomes increasingly important for getting policy outcomes to reflect popular sentiment.\textsuperscript{89}

Even if some imagined ideal of districting could maximize the likelihood of a majoritarian outcome, lack of congruence with this standard could still arise by chance and small variations in opinion. In 2012, if a few thousand voters in Arizona had cast their ballots for a Republican instead of a Democrat in the First or Second District, the delegation would have been, like the state’s popular vote, majority Republican.\textsuperscript{90} Thus, antimajoritarian outcomes are not

\textsuperscript{86} A failure rate of 2 out of 34, or 6%, may be considered acceptable, when one considers the following comparison: in the history of the United States, the popular-vote winner has failed to win the presidency in 4 out of 57 elections, a 7% rate. See \textit{United States Presidential Election Results}, DAVE LEIP’S ATLAS U.S. PRESIDENTIAL ELECTIONS (2012), http://uselectionatlas.org/RESULTS (to locate, search presidential election results by year and compare popular vote to electoral vote). However, presidential elections rely on fixed state boundaries. Maintaining representative performance in legislative elections is vulnerable to variations in where and how district boundaries are drawn.

\textsuperscript{87} See HAAS, supra note 29, at 3, 31, 46, 53, 66.


\textsuperscript{89} See Delia Baldassarri & Andrew Gelman, \textit{Partisans Without Constraint: Political Polarization and Trends in American Public Opinion}, 114 AM. J. SOC. 408, 423-26 (2008) (showing that across a range of economic, civil rights, and moral issues, correlations between issue partisanship and party identification are positive and increasing over time).

always accurate indicators of partisan maneuvering. Furthermore, a simple majoritarian standard is incomplete because it only addresses the issue of whether seats or votes fall above or below a 50% threshold. For example, if a party receives 51% of the vote, receiving 55% or 80% of the seats are both majoritarian outcomes but might be viewed quite differently.

A statistical approach is needed to distinguish what degree of inequity is allowable. I use natural variation and basic concepts of statistics to build three tests for state-level partisan gerrymandering. My approach allows the user to consider conceptual subtleties and at the same time obtain unambiguous judgments without need for elaborate computation using methods whose details have either not been widely adopted by political science researchers, and/or found by courts not to be persuasive in the outcome. I hope that a more straightforward approach may meet wide approval and serve as a universal tool to assess claims of partisan gerrymandering objectively.

II. Quantitatively Analyzing the Effects and Intents of Partisan Gerrymandering

The Vieth plurality referred disparagingly to the concept of fairness as “flabby.” Quantitative approaches open the possibility of formulating a more muscular definition. This Article will provide methods to identify partisan unfairness at the statewide level, resulting in proposed standards for partisan gerrymandering that do not require the drawing of hypothetical maps.

To establish statistical tests, it is first important to examine past patterns of gerrymandering. I use several well-known examples to illustrate two analyses. The Analysis of Effects (in Subpart A) uses computer simulations to quantify the effects of gerrymandering. This analysis of effects can then be used as independent validation for the Analysis of Intents (in Subpart B), which identifies when win margins have been arranged to give a systematic pattern of reliable wins. The Analysis of Intents, which reflects the intent of the redistricting party, can be done using a hand calculator easily and rapidly.

This approach recalls Justice Kennedy’s statement that "new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties. That would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards.”

92. Id. at 312-13 (Kennedy, J., concurring in the judgment).
A. Analysis of Effects: What Is an Appropriate Range of Seats for a Given Share of Votes?

1. Distinguishing partisan distortion from Voting Rights Act section 2 constraints

Although partisan gerrymandering is considered justiciable, another practice that uses similar districting methods is permitted and even mandated under section 2 of the Voting Rights Act—the establishment of districts in which an ethnic minority constitutes a majority of the district’s inhabitants. These “majority-minority” districts are constructed to ensure that the interests of identified subgroups are represented. When minorities constitute less than fifty percent of a state’s population, they can end up on the losing side of every election. To counteract this risk, majority-minority districts are constructed to cluster groups with shared interests.

This dual use of district-drawing methods opens the challenge of how to construct an analysis that identifies partisan gerrymandering as anomalous, but not single districts that are drawn to create ability-to-elect districts such as majority-minority districts. Such an analysis will require the evaluation of groups of districts at once. Existing doctrine may provide some guidance.

Among the standards for the proper establishment of majority-minority districts is the concept that majority-minority districts should comprise a fraction of all districts that does not exceed the proportion of the minority population. Under existing precedent, the “no-more-than-proportional” concept contributes to the criteria for evaluating districting schemes. Where minority representation is concerned, the criteria identify rough proportionality as a relevant factor in evaluating the fairness of a districting plan. Under that standard, the Court has held “that no violation of § 2 can be found here, where, in spite of continuing discrimination and racial bloc voting, minority voters form effective voting majorities in a number of districts roughly proportional to the minority voters’ respective shares in the


96. See 52 U.S.C. § 10301(b) (“Nothing in this section establishes a right to have members of a protected class elected in numbers equal to their proportion in the population.”).

voting-age population. For example, if a minority group with twenty percent of a state’s eligible population is able to elect representatives in twenty percent of a state’s districts, this argues against violation of the Gingles criteria.

The idea underlying the Gingles criteria can be used to address the question of appropriate representation by political parties. I suggest that a redistricting plan is acceptable if it moves the seats-to-votes outcome toward partisan proportionality (eu-proportionality) as measured by prevailing national standards, and unacceptable if it moves the outcome away from proportionality (dys-proportionality) beyond the zone of chance. This standard can be understood at a glance using a graph (Figure 1) that I term a “representation plot,” or alternatively a “bowtie plot,” where eu-proportional outcomes are “inside the bowtie.” Since dys-proportional outcomes are a major result of partisan gerrymandering, a standard should distinguish between eu-proportionality and dys-proportionality.

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98. 512 U.S. at 1000.
99. See Gingles, 478 U.S. at 74-77 (describing near-proportional legislative representation of black voters as evidence of their ability to elect their preferred representatives).
100. In this plot, the black line indicates proportionality and is a straight line drawn from zero vote share and zero seat fraction to 100% vote share and 100% seat fraction. The seats/votes curve is calculated by resampling to build “fantasy delegations,” see infra Part II.A.3, and is approximated by the mathematical function that is the area under a bell-shaped curve whose average is 50% vote share, and whose standard deviation is 14% vote share.
I note that the eu-proportionality concept specifically does not imply the establishment of proportional representation, a rule that is not to be found in the Constitution or districting law. For example, in the domain of racial criteria, the *Gingles* precedent says that proportionality is neither mandated nor is it a safe harbor, but rather that proportionality is important evidence of fairness. And, as stated in the Introduction, proportionality does not naturally arise in a single-member district system. Single-member districts usually
generate outcomes in which a party’s share of seats tends to exceed its proportion of popular support. Instead, the eu-proportionality concept relies on the idea that some deviations from an average seats-to-votes relationship are beneficial for representation, whereas other deviations are detrimental. Good districting seeks to establish “fair and effective representation for all citizens.” The concept that deviations toward proportionality are good encompasses a wide range of concepts that includes: (1) establishing appropriate levels of representation for minority groups (in other words, the Gingles criteria); (2) allowing the possibility that, like a racial group, a political party with considerably less than fifty percent support might permissibly have enhanced representation relative to what would be predicted from national seats/votes relationships, but that reduced representation is impermissible; and (3) setting reasonable limits to how much enhancement from (2) is allowed. In this way, the Platonic ideal of proportionality does not set a specific goal, but instead defines a direction of acceptable deviation. It is simple to state, it is flexible, and it contains many permissible outcomes.

2. Defining the zone of chance

In addition to defining desirable and undesirable directions, a standard for partisan gerrymandering requires a method for determining whether a change could have arisen as part of normal variation in districting as practiced across the United States. I use the rules of probability to (1) describe that variation, (2) establish what the range of possible outcomes is, and (3) formulate a rule for identifying situations in which a state’s new districting scheme has departed sufficiently from normal practice.

Faulty bright-line standards, such as a majoritarian standard, can be repaired by identifying a “zone of chance,” which I define as the range of outcomes that could have arisen, without deliberate planning, from variations in how districts are drawn. I calculate zones of chance for (1) the number of

101. Proportional representation is achieved only in systems where it is enforced specifically and directly. For example, in Israel, members of the national legislative body, the Knesset, are assigned so that the number of a party’s seats is proportional to the fraction of its popular vote. Basic Law: The Knesset, 5718, § 4, 180 LSI 18 (1987-2003). Such a system embodies a legislature-centered form of the “one person, one vote” principle: Each citizen’s party preference is reflected proportionally at the national level.


103. See Wang, supra note 29.

104. The zone-of-chance concept is a way to express the concept of significance testing in statistics. Statisticians calculate how far a measurement, such as the number of seats won by a party in a given election, is likely to stray from the expected average. In this Article, I define the zone of chance as a region within which chance outcomes would fall 95% of the time and outside the region 5% of the time. Statistics texts refer to this as a "p<0.05" or "α<0.05" standard. See Lowry, supra note 61; see also Wang, supra note 29.
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seats won in an election for any given statewide division of popular vote, and (2) the pattern of voting outcomes across districts.

The zone-of-chance approach recalls Justice Kennedy’s statement that “new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties. That would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards.” At the same time, I also take advantage of longstanding statistical tests whose history assures their mathematical rigor. The use of statistical tests also allows judges to evaluate evidence more directly, with less need for assistance from external experts.

To understand the zone-of-chance concept, it is helpful to start by considering a case that is mathematically simple and does not require computer simulation—equally matched parties.

As pointed out in the plurality opinion in Vieth v. Jubelirer, any districting scheme contains the possibility that a majority of votes will, by chance, lead to a minority of seats. To explore this concern, it is informative to calculate the exact probability that such a deviation could occur in the absence of intentional partisan districting. The calculation is simplest when the two-party popular-vote share (defined as the fraction of the top two parties' popular vote won by one party) is close to fifty percent for each party, and individual districts are closely divided. In this circumstance, Party A's seat-share for a random partitioning of N districts is on average N/2, and the probability of Party A winning a particular district is 0.5. The actual number of districts won will vary, in the same way that a series of coin tosses are not guaranteed to yield equal numbers of heads and tails. We can calculate the standard deviation, a statistical quantity that is useful because the outcome will be within one standard deviation of the average about two-thirds of the time; thus, outcomes within this range would be fairly unsurprising. And if the


106. It must be noted that the simplified formula for sigma described in this paragraph is a substantial overestimate of real-life situations because districting generates a mixture of more and less closely-contested districts, and only close contests contribute to uncertainty. To estimate the true value of sigma, which is typically smaller, a more sophisticated approach is required, as detailed in Part II.A.3 below.

107. For example, if all N races are perfect toss-ups, then they behave like coin tosses, and according to the laws of probability, the standard deviation of the seat outcome—a measure of variation often referred to as sigma, or σ—is 0.5√N. Thus if political parties A and B compete in a state that is composed of sixteen congressional districts, all of which are closely contested, then each party can expect to get eight seats on average. Sigma for the specific case of all-close-races is 0.5√16 = 2 seats, suggesting that each party would typically get six to ten seats. For an approximate formula that applies to a wider range of situation, see infra note 117.
vote share is almost exactly fifty percent, then outcomes will give a majority to the minority side close to half of the time.

To generalize the zone-of-chance calculation, I use computer simulation to calculate the standard deviation, which in turn establishes a zone of chance, for fractions of the vote other than 0.5. I use existing districts in the year under examination as a source of information about how vote totals in districts may vary. The inputs to the calculation are the congressional vote totals for the state under examination and national district-by-district congressional results from the same year. This process escapes the burden of drawing boundaries, which requires the researcher to apply her standards about “good districting.” This calculation will yield both a general seats/votes relationship and a statistical confidence interval (i.e., zone of chance) for the range of outcomes that could be expected in the absence of directed partisan intent. The zone of chance provides an answer to the question whether a set of election outcomes has deviated sharply from national standards.

3. National districting patterns can be used to identify a natural seats/votes relationship

Computer simulations can be used to ask a simple question: If a given state’s popular House vote were split into differently composed districts carved from the same statewide voting population, what would its congressional delegation look like? The answer allows the definition of a range of seat outcomes that would arise naturally from districting standards that are extant at the time of the election in question.

It is possible to calculate each state’s appropriate seat breakdown—in other words, how a congressional delegation would be constituted if its districts were not contorted to protect a political party or an incumbent. This is done by randomly selecting combinations of districts from around the United States that add up to the same statewide vote total for each party. Like a fantasy baseball team, a delegation put together this way is not constrained by the limits of geography. On a computer, it is possible to create millions of such unbiased delegations in short order. In this way, one can ask what would happen if a state had districts whose distribution of voting populations was typical of the pattern found in rest of the nation.108 Because this approach uses existing districts, it uses as a baseline the asymmetries that are present nationwide.109 Indeed, the average result of these simulations approximates a

108. This can be done by using all 435 House race outcomes. For a state X with *N* districts, you would calculate the total popular vote across all *N* districts, then pick *N* races from around the country at random and add up their vote totals. If their vote total matches X’s actual popular vote within 0.5%, score it as a comparable simulation. See, e.g., Wang, supra note 27.

109. It is possible to explore the properties of this simulation procedure by giving it a variety of hypothetical nationwide distributions of districts as starting data. These

*footnote continued on next page*
“natural” seats/votes relationship that can be defined with mathematical rigor and exactitude. In short, these simulations detect distortions in representativeness in one state, relative to the rest of the nation.

Using a standard ThinkPad X1 Carbon laptop computer equipped with the mathematical program MATLAB, simulation code\textsuperscript{110} can perform one million simulations for a state in less than twenty seconds. Figure 2 shows one thousand such “simulated delegations” for the state of Pennsylvania, along with the actual outcome in gray. The solid curve defines a mathematically expected average seats/votes relationship.

\textsuperscript{hypothetical scenarios reveal that the “fantasy delegation” procedure has important features that are required of a detector of partisan asymmetry. First, for a symmetric distribution of congressional districts, i.e., a scenario in which Democrat-dominated districts are no more packed than Republican-dominated districts, fantasy delegations are typically majoritarian, awarding more representatives to the party that receives more votes. Second, the fantasy delegations have the same natural variation in partisan composition as the actual nationwide distribution of state delegations, as measured by standard deviation. Third, when the nationwide distribution of districts has asymmetry, for instance containing a number of districts that are very packed with one party (as is the case in real life for Democrats), the fantasy delegations show a bias toward the other party, a phenomenon that is well analyzed. See Chen & Rodden, \textit{Unintentional Gerrymandering}, supra note 75.

\textsuperscript{110} The MATLAB software is available at Sam Wang, \textit{Gerrymandering}, GITHUB, https://github.com/SamWangPhD/gerrymandering (last visited June 6, 2016).
Figure 2
Simulated Pennsylvania House Delegations

Each point indicates one hypothetical delegation composed of eighteen House districts drawn at random from the national House election of 2012. One thousand simulations are shown. The solid curve indicates the average seats/votes curve and the dashed line indicates proportionality, both as defined in Figure 1. The gray shaded data point indicates the actual outcome, which falls in a zone of dys-proportionality, “outside the bowtie.”

It is apparent that most possible redistrictings would have resulted in a more equitable congressional delegation. For outcomes with the same popular-vote split (50.7% Democratic, 49.3% Republican), one million simulations gave a median result of eight Democratic, ten Republican seats (an average of 8.5 Democratic seats). The actual outcome was five Democratic, thirteen Republican; however, only 0.2% of the simulations with the same popular vote (i.e., 50.7% Democratic) led to such a lopsided (or a more lopsided) split favoring Republicans.

Pennsylvania is known to have been targeted by the Republican State Legislative Committee’s Redistricting Majority Project (REDMAP), a multiyear effort to facilitate and carry out aggressive redistricting after the
A similar computational analysis of all fifty states can be done to test if additional REDMAP states show statistical anomalies.

For all fifty states, Figure 3 is calculated using the vote outcomes of non-extreme states (shaded in light gray) to feed the simulations. These results coincide strongly with targeted partisan redistricting efforts and are highly unlikely to have arisen by chance. White shading indicates Republican Party control over redistricting, dark gray indicates Democratic Party control, and black indicates nonpartisan commission (AZ, Arizona) or a court-ordered map (TX, Texas). Out of ten states with extreme outcomes, eight favored the party that controlled the process, and none worked against the party in control. Indeed, the extreme cases include all states with single-party control that have been mentioned on a redistricting watchlist published in 2011 by the Washington Post.

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112. Statewide vote totals may include some races that are uncontested. In these districts, it is not known how the voters would have decided if they had an alternative choice. In order to address this, it may also be necessary to assign those voters assuming a split other than 100%-0%. One established approach is to assume a 75%-25% split. See Andrew Gelman & Gary King, A Unified Method of Evaluating Electoral Systems and Redistricting Plans, 38 AM. J. POL. SCI. 514, 550 (1994). Generally, this assumption does not affect the outcome of the tests in this Article.

113. See Dickinson, supra note 111; Giroux, supra note 16; Pierce et al., supra note 111.

114. In Arizona, small shifts in voting in either the second or ninth district would have altered the overall outcome to near-neutrality. See Ariz. Sec’y of State, supra note 90.

"R+" indicates that the actual outcome was more favorable to Republicans than random resampling from national races. "D+" indicates that the actual outcome was more favorable to Democrats. Color shading for discrepancies greater than 1.2 seats indicates who controlled redistricting: white for Republicans, black for Democrats, and grey shading for mixed control (Arizona was redistricted by a nonpartisan commission and Texas was redistricted first by Republicans, and then changed by court order).

In Part II.B below, I develop an analysis of intent that again uses the zone-of-chance concept. There, as here, the standard deviation, sigma (σ), will be used as a yardstick of deviations from the average expected outcome. As before, the general idea is that an average outcome only reflects one point in a range of outcomes, and the standard deviation is necessary to define a zone of chance. Generally speaking, for a bell-like curve, which these simulations approximately follow, a difference of 1.6 standard deviations or more occurs by chance in five percent of cases. Five percent is a common threshold for determining statistical significance. The standard deviation is a handy and universal reference measure for detecting extreme outcomes, and it applies to all the analyses and tests in this Article. For convenience of notation in the

116. A difference of Delta = 1 or more in a dys-proportional direction occurs in approximately 16% of cases. A difference of Delta = 2 or more occurs in approximately 2.3% of cases. A difference of Delta = 3 or more occurs in approximately 0.13% of cases. These values are for Analysis 1, which uses a bell-shaped curve, the usual assumption for statistical testing. Analyses 2 and 3 use the t-distribution, which gives slightly different values. I define the quantity Delta (Δ) as the difference from average expectations, divided by sigma.
tables that follow, I define the quantity Delta ($\Delta$) as the difference from average expectations, divided by sigma.

Table 1 shows states for which the partisan discrepancy was greater than one sigma in 2012. For comparison, discrepancies for the same states are shown for 2010 and 2014. Simulation-based values for sigma are given in the columns labeled “SD (sigma).”

117. These values are approximated reasonably well by the formula $\sigma = 0.52 \times \sqrt{(s + (N - s) / N)}$, where $N$ is the number of a state’s congressional districts and $s$ is the average number of seats won in that state by either major party in computer simulations. The principal difference from the “all toss-ups” example is the appearance of a factor of 0.52, which arises from the fact that some districts are competitive, and some are not; this factor fell within a narrow range of 0.50-0.53 between 2008 and 2014.
# Table 1

<table>
<thead>
<tr>
<th>State</th>
<th>Total Seats</th>
<th>Democratic Seats</th>
<th>Democratic Vote Share</th>
<th>Simulated Average (Sigma)</th>
<th>Difference in SD</th>
<th>Total Seats</th>
<th>Democratic Seats</th>
<th>Democratic Vote Share</th>
<th>Simulated Average (Sigma)</th>
<th>Difference in SD</th>
<th>Total Seats</th>
<th>Democratic Seats</th>
<th>Democratic Vote Share</th>
<th>Simulated Average (Sigma)</th>
<th>Difference in SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arizona</td>
<td>8</td>
<td>46.74%</td>
<td>3.18</td>
<td>0.78</td>
<td>R by 0.2</td>
<td>9</td>
<td>45.60%</td>
<td>5</td>
<td>2.96</td>
<td>0.76</td>
<td>D by 2.7</td>
<td>49.17%</td>
<td>3</td>
<td>2.94</td>
<td>0.75 D by 1.4</td>
</tr>
<tr>
<td>Florida</td>
<td>25</td>
<td>42.53%</td>
<td>7.38</td>
<td>1.31</td>
<td>R by 1.1</td>
<td>27</td>
<td>50.00%</td>
<td>10</td>
<td>11.73</td>
<td>1.33</td>
<td>R by 1.3</td>
<td>49.02%</td>
<td>10</td>
<td>12.32</td>
<td>2.59 R by 0.9</td>
</tr>
<tr>
<td>Illinois</td>
<td>19</td>
<td>54.96%</td>
<td>11.15</td>
<td>1.23</td>
<td>R by 1.6</td>
<td>18</td>
<td>55.40%</td>
<td>12</td>
<td>10.04</td>
<td>1.11</td>
<td>D by 1.8</td>
<td>51.42%</td>
<td>10</td>
<td>9.14</td>
<td>1.12 D by 0.8</td>
</tr>
<tr>
<td>Indiana</td>
<td>9</td>
<td>42.04%</td>
<td>2.53</td>
<td>0.78</td>
<td>D by 0.6</td>
<td>9</td>
<td>45.80%</td>
<td>2</td>
<td>3.02</td>
<td>0.76</td>
<td>R by 1.3</td>
<td>38.90%</td>
<td>2</td>
<td>2.03</td>
<td>0.72 R by 0.04</td>
</tr>
<tr>
<td>Maryland</td>
<td>8</td>
<td>63.42%</td>
<td>6.13</td>
<td>0.78</td>
<td>R by 0.2</td>
<td>8</td>
<td>65.46%</td>
<td>7</td>
<td>6.11</td>
<td>0.72</td>
<td>D by 1.2</td>
<td>58.14%</td>
<td>7</td>
<td>5.13</td>
<td>0.76 D by 2.4</td>
</tr>
<tr>
<td>Michigan</td>
<td>15</td>
<td>47.87%</td>
<td>6.35</td>
<td>1.07</td>
<td>R by 0.3</td>
<td>14</td>
<td>52.70%</td>
<td>5</td>
<td>6.97</td>
<td>0.98</td>
<td>R by 2.0</td>
<td>50.88%</td>
<td>5</td>
<td>6.96</td>
<td>0.98 R by 2.0</td>
</tr>
<tr>
<td>North Carolina</td>
<td>13</td>
<td>42.47%</td>
<td>4.30</td>
<td>0.94</td>
<td>D by 0.2</td>
<td>13</td>
<td>50.90%</td>
<td>4</td>
<td>5.94</td>
<td>0.95</td>
<td>R by 2.1</td>
<td>45.31%</td>
<td>3</td>
<td>4.89</td>
<td>0.92 R by 2.1</td>
</tr>
<tr>
<td>Ohio</td>
<td>18</td>
<td>44.75%</td>
<td>6.26</td>
<td>1.14</td>
<td>R by 1.1</td>
<td>16</td>
<td>47.90%</td>
<td>4</td>
<td>6.48</td>
<td>1.08</td>
<td>R by 2.4</td>
<td>40.94%</td>
<td>4</td>
<td>4.40</td>
<td>0.98 R by 0.4</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>19</td>
<td>48.41%</td>
<td>8.27</td>
<td>1.21</td>
<td>R by 1.1</td>
<td>18</td>
<td>50.70%</td>
<td>5</td>
<td>8.14</td>
<td>1.01</td>
<td>R by 2.9</td>
<td>46.07%</td>
<td>5</td>
<td>7.06</td>
<td>1.09 R by 1.9</td>
</tr>
<tr>
<td>Texas</td>
<td>32</td>
<td>37.51%</td>
<td>5.69</td>
<td>1.30</td>
<td>D by 1.0</td>
<td>36</td>
<td>39.90%</td>
<td>12</td>
<td>8.68</td>
<td>1.18</td>
<td>R by 2.3</td>
<td>39.35%</td>
<td>11</td>
<td>8.65</td>
<td>1.43 D by 1.6</td>
</tr>
<tr>
<td>Virginia</td>
<td>11</td>
<td>44.94%</td>
<td>3.87</td>
<td>0.89</td>
<td>R by 1.0</td>
<td>11</td>
<td>49.00%</td>
<td>3</td>
<td>4.56</td>
<td>0.86</td>
<td>R by 1.8</td>
<td>44.84%</td>
<td>3</td>
<td>4.02</td>
<td>0.84 R by 1.2</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>8</td>
<td>45.40%</td>
<td>2.91</td>
<td>0.77</td>
<td>D by 0.1</td>
<td>8</td>
<td>50.76%</td>
<td>3</td>
<td>3.64</td>
<td>0.73</td>
<td>R by 0.9</td>
<td>47.20%</td>
<td>3</td>
<td>3.36</td>
<td>0.73 R by 0.5</td>
</tr>
</tbody>
</table>

For 2010, 2012, and 2014, one million simulations were done for each state, resampling was done from nationwide House election returns for that year. The “SD (sigma)” column indicates the value of σ calculated from the simulations. Boldface text indicates values of Delta (difference between simulation and actual results exceeding one times σ) favoring either party. Boldface underline indicates differences exceeding two times σ. Note the persistence of effects in 2014.
Five states showed deviations that were greater than one sigma and less than two sigma: Florida, Illinois, Indiana, Maryland, and Virginia. Six more states showed a deviation exceeding two sigma: Arizona, Michigan, North Carolina, Ohio, Pennsylvania, and Texas. Of these eleven states, REDMAP’s redistricting efforts are known to have targeted five: Indiana and all four Republican-controlled states with two-sigma discrepancies, namely Michigan, North Carolina, Ohio, and Pennsylvania. Of the remaining greater-than-two-sigma states, a fifth state, Texas, was redistricted by Republicans but showed a discrepancy favoring Democrats. A sixth state, Arizona, was redistricted by an independent commission and favored Democrats.

Of these six states, I briefly describe three cases of special interest: California, Texas, and Florida.

**California.** California is worth mentioning as a counterexample to the imbalanced states shown above. California was redistricted by an independent commission. In 2012, the California House popular vote was 62% Democratic, resulting in 38 out of 53, or 72%, Democratic seats. However, the average simulated delegation was also 72% Democratic. Thus, election results in California exactly meet the expectations that arise from nationwide districting patterns.

**Texas.** Although the resampling simulations are a powerful and sensitive measure, the case of Texas demonstrates how examination of additional factors can be necessary. Before the 2012 election in Texas, a complex series of legal battles culminated in a court-ordered redistricting plan and a congressional

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122. HAAS, supra note 30, at 72.

123. A theoretical symmetric distribution of districts would, on average, give a delegation that is 79% Democratic. For a symmetrically distributed distribution of districts whose two-party vote share has standard deviation SD, the expected fraction of seats S for a given vote share V is normcdf((V-0.5)/SD), where normcdf is the integral of a bell-shaped normal curve with mean 0 and width parameter 1. For non-dys-proportional states in 2012, SD = 0.15, comparable to longstanding findings for seats/votes curves. See GRAHAM GUDGIN & PETER J. TAYLOR, SEATS, VOTES, AND THE SPATIAL ORGANISATION OF ELECTIONS 20-31(1979).

election outcome in which over 60% of Texas voters voted for Republicans, resulting in 24 Republican seats out of 36 total. From a statistical standpoint, this was an underperformance for Republicans, who in a simulation would have won over 28 seats on average—a discrepancy of Delta = 2.3 times sigma, which is outside the zone of chance, and therefore a statistically significant deviation. One major factor contributing to this discrepancy was the presence of Hispanic majorities in 9 districts, 6 of which elected Democratic congressmen. These majority-minority districts, which have special status under the Voting Rights Act of 1965, reflect the growing Hispanic population in Texas, which as of the 2010 census constituted 38% of Texans. Democrats won approximately 40% of the statewide two-party popular vote and won 12 out of 36 seats, or 33% of seats. Because this change is in the direction of proportionality compared with typically occurring seats-votes curves, it is eu-proportional. The number of majority-minority districts (which usually favor Democrats) falls within the Gingles criteria. Thus, the final outcome in Texas in 2012 favored the partisan minority for mandated race-based reasons, and because it is eu-proportional, would not be grounds for further action.

Florida. In this case, where the value of Delta is between one and two, a similar but statistically stronger answer is given by a map-drawing approach. Chen and Rodden took a geographically intensive approach, drawing districts using automated rules of contiguity and community preservation, and implemented these rules thousands of times through detailed computer simulations. They found that Florida’s 2010 redistricting scheme was more favorable to Republicans than over 99% of their simulations, indicating that the Florida legislature applied an approach that led to a more partisan outcome than Chen and Rodden’s rules would support. Geographic considerations are among the principles of districting mandated by the Constitution of the State of Florida, which also allows for judicial review by the Florida Supreme Court. In July 2015, that court replaced the map to comply with the state constitution.

125. See HAAS, supra note 30, at 57-59.
127. Id.
130. Chen & Rodden, Cutting Through the Thicket, supra note 75, at 335-38.
131. Id. at 338.
132. FLA. CONST. art. III, §§ 16(c), 20-21.
133. League of Women Voters of Fla. v. Detzner, 172 So. 3d 363, 371-72 (Fla. 2015).
Nationwide, repairing the one-sigma and greater Republican-redistricted states (seven in all) would lead to an average swing of approximately twenty-eight seats (an average of 27.7) toward Democrats; repairing the two Democratic-redistricted states, Illinois and Maryland, would lead to an average swing of 5.7 seats toward Republicans. Therefore, based on these measures, Republican gains in 2012 from aggressive redistricting (28 seats) were nearly five times the advantages gained by Democrats from the same process (6 seats). This sharp asymmetry coincides with a period during which state legislative processes have come increasingly under single-party control. Changes between decadal redistrictings favored Republicans, who controlled 13 state capitals in 2002, rising to 24 state capitals in 2012. During that same interval, Democrats went from controlling 8 state capitals to controlling 13 state capitals. Thus the potential for partisan control of districting has increased for both major parties, with a greater advantage for the Republican Party.

4. What accounted for the antimajoritarian outcome of 2012?

With these analytical tools in hand, it is now possible to calculate the total effect of asymmetric partisan districting on the national House elections of 2012. The outcome was a 33-seat margin of control, with 234 Republican, and 200 Democratic seats. Applying party-neutral standards to the 7 Republican-controlled states and 2 Democratic-controlled states would have given an average margin that was 22 seats smaller, or 212 Democrats and 223 Republicans. Because of the uncertainty contained in this analysis (the range of outcomes within two sigma of the average was 206 to 218 Democratic seats), it is just within the range of possibility that without partisan asymmetry, Democrats might have taken control of the chamber.

Republicans have a second advantage, one that arises from population clustering. Because this simulation-based analysis uses existing national districts, it includes effects of population clustering. It is possible to quantify the net impact of population clustering, which facilitates the drawing of districts that are heavily tilted toward Democrats.


135. Davey, supra note 134; State Government Control Since 1938, supra note 134.

136. Davey, supra note 134; State Government Control Since 1938, supra note 134.

137. HAAS, supra note 30, at 74.

138. Chen & Rodden, Unintentional Gerrymandering, supra note 75, at 260-64 (calculating the biases associated with simulated redistricting using compactness principles across the fifty states).
In the original simulations, states where I did not find dys-proportionality had a two-party vote share of 50.7% for Democrats and 180 out of 363 seats. I then calculated the expected share of seats if district-by-district vote shares were perfectly symmetrically distributed. Such symmetry of population patterns predicts that a 50.7% vote share would lead to Democrats winning 51.8% of seats, or 188 seats, 8 seats more than the real-population-based simulation. Scaling this up to all 435 seats, this suggests that Republicans won 9 or 10 seats in non-dys-proportional states more than they would under symmetric population patterns; the swing, defined as margin between the parties, is twice as large—18 to 20 seats. This 18-to-20 seat swing effect across all fifty states is smaller than the 22-seat effect of partisan dys-proportionality in just nine states. Therefore although a considerable deviation from natural seats/votes relationships is driven by political geography, an even larger total effect arises from political motivations and actors during the legislative process in just a handful of states.

In summary, the effects of partisan redistricting exceeded the amount of asymmetry caused by natural patterns of population. Together, gerrymandering and population clustering are more than enough to account for the fact that in 2012, Democrats won the House popular vote but Republicans ended up in control of the chamber.

B. Analysis of Intents: Voter Packing by Intentional Gerrymandering and Self-Association

The Analysis of Effects (discussed in Part II.A above) established a method for identifying states in which voter preferences lead to representation that is anomalous relative to national norms. Without gerrymandering, these anomalies could be rectified through the ballot box: if election outcomes shift sufficiently, legislators can be voted out, thus bringing outcomes more in line with the popular will. As an example of how gerrymandering vitiates this mechanism, the election of 2014 heralded a "wave year" in which Republicans won the national popular vote by 5.9%, in sharp contrast to the Democratic popular vote win of 2012. However, in the twelve states in Table 1, Republicans gained control of only 5 of 187 Democratic seats. This small

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139. See GUDGIN & TAYLOR, supra note 123, at 20-31.
140. This effect is consistent with previous work. See Stephanopoulos & McGhee, supra note 17, at 873 fig.5.
142. Compare id. at 2, 9, 13, 19, 21, 36, 37, 40, 44, 48, 50 (giving House election outcomes in 2012 for the states listed in Table 1 above), with HAAS, supra note 30, at 2, 12, 18, 27, 30, 46, 47, 52, 63, 65 (giving House election outcomes in 2014 for the states listed in Table 1 above).
change indicates that representatives in these states were largely insulated from a large swing in opinion from 2012 to 2014. Considering the strength of partisan gerrymandering in 2012, the smallness of this change means that Republicans reaped most of their electoral gains two years earlier than their popular support would have merited.

The Analysis of Intents below presents a way to identify asymmetric reductions in the ability of legislative elections to respond to changes in voter opinion. It therefore can be used to measure a principal effect of partisan gerrymandering: reduction in the overall responsiveness of races across a state. I propose that such a pattern is indicative of the intentions of the entity that drew the district lines.

1. Distinctive patterns of win and loss margins arising from partisan gerrymandering and voter self-association

Partisan redistricting procedures create a characteristic lopsided pattern of election results that can be used to identify when packing is likely to have occurred. State-level gerrymandering is more elaborate than single-district gerrymandering and relies on a two-part strategy. First, as before, map-drawers will cram “voters likely to favor [their] opponents into a few throwaway districts where the other side will win lopsided victories, a strategy known as ‘packing.’”143 Second, map-drawers will draw the remaining, more numerous districts using boundaries to yield more narrowly won victories.144 In this process, the critical requirement is asymmetry: the opposing party’s voters must be more tightly packed than one’s own voters.145 The net result is an increased likelihood of unrepresentative outcomes.

I examine lopsided patterns in gerrymandered states and compare them to nongerrymandered states: this provides a comparison of the effects of partisan gerrymandering with the effects of population variations and less-partisan districting. This analysis will be used as the basis for Test 2 in Part IIIA, an index of gerrymandering that depends directly on the partisan redistricter’s desired goal: the packing of opponents, as measured by election returns.

Gerrymandered districts show a distinctive pattern of lopsided votes (Figure 4). Figure 4a shows a histogram of two-party vote share for 2012 House

143. Wang, supra note 29.
144. LEVITT, supra note 11, at 58.
145. Because members of both major parties get packed into districts in a partisan gerrymander, individual members of the opposing party may acquiesce or even be complicit in the process. See, e.g., League of United Latin Am. Citizens (LULAC) v. Perry, 548 U.S. 399, 418 (2006) (“[A] number of line-drawing requests by Democratic state legislators were honored.”). In other words, a single-district gerrymander can favor one party even as a partisan gerrymander favors the other party. For this reason, the use of intent as a standard for gerrymandering should distinguish between district-level and party-level motivations.
districts that were asymmetric in favor of Republicans. In this histogram, two peaks are apparent: a narrow peak centered near a 40% Democratic vote share and a broader peak centered near a 30% Republican vote share (indicated on the histograms by a 60% to 80% Democratic vote share). Both of these peaks are sufficiently prominent that they can also be seen in a histogram drawn using all states nationwide (Figure 4a). The peaks are considerably more prominent when the histogram includes only Republican-favoring states (Figures 4b and 4c) or Democrat-favoring states (Figure 4d).

**Figure 4**
District-By-District Histograms of 2012 House Election Results
(a) All districts, showing one peak with close Republican wins and one peak with lopsided Democratic wins. These peaks persist in (b) districts from states showing an overall advantage to Republicans of 2 times sigma or greater, based on resampling simulations; and (c) districts with a Republican advantage of 1 to 1.9 times sigma; but are shifted or absent in (d) Democratic-advantaged states. Shown for comparison are (e) states with urbanized populations, defined as greater than 1 million people in cities larger than 250,000, in the 2010 census; (f) districts with more than 1000 persons per square mile; (g) the same districts as (f) but with the districts in (b) and (c) removed; and (h) the same districts as (g) but with the districts in (f) removed. The light gray zone indicates Republican wins, and the dark gray zone indicates Democratic wins.

However, as stated in Part II.A.4 above, voter packing can be asymmetric simply by virtue of the fact that voters arrange themselves in a manner that is not symmetric. Therefore any measure of gerrymandering-based packing must be done relative to a baseline of how voters “pack themselves.” Specifically, it has been suggested that structural factors such as concentration of Democrats in urban areas may have a greater effect than partisan redistricting.146 I now show how these two effects are manifested in district-level outcomes. Since both real packing by redistricters and virtual packing by structural factors are likely to have similar manifestations, they can be examined using the same statistical tools.

146. Chen & Rodden, Unintentional Gerrymandering, supra note 75, at 241.
2. Gerrymandering emulates and amplifies the representational consequences of urbanization

The establishment of competitive districts is made difficult by the fact that voters often choose to live near others of similar ethnic, religious, secular, and political affiliation.\textsuperscript{147} Such self-selection is visible in urban regions that vote overwhelmingly for Democrats and rural regions that vote overwhelmingly for Republicans. If natural population clustering favors increased Republican representation, then the distribution of vote share in urbanized districts should resemble that of Republican-gerrymandered states. Such a pattern is not apparent in high-population-density states (Figure 4e). However, urbanized districts (Figure 4f), defined as those with population density greater than 1000 persons/square mile, show both peaks, but with more emphasis on the high-Democratic-vote share peak. This pattern is visible even when putatively gerrymandered states (favoring both Democrats and Republicans) are omitted from the histogram (Figure 4g).

Gerrymandering makes use of existing urbanization. In Republican-gerrymandered states, non-urbanized districts (Figure 4b and 4c) are dominated by Republican-packed districts, demonstrating that redistricters who seek a Republican advantage do so by creating numerous districts that avoid urban regions. Once Republican gerrymanders and urbanized areas are omitted, a histogram of the remaining congressional districts no longer has two peaks (Figure 4h). Democratic gerrymanders can achieve a converse advantage by carving out slices of urbanized areas and combining them artfully with more rural areas to create small but secure Democratic wins.

Although the representational effects of voter migration into urban communities are similar to the effects of partisan gerrymandering, the interpretations of the two phenomena are quite different. Voters who arrange themselves in this manner are voluntarily arranging themselves so that their representatives are at little risk of being turned out of office. In the case of partisan gerrymandering, voters are placed into political affiliation with one another—but without the consent of the citizens involved. Such a pattern contradicts the saying that "voters should choose their representatives, [and] not the other way around."\textsuperscript{148} Gerrymandering thus penalizes voters based on their publicly available information, including partisan loyalty, which is present in census data and commercial redistricting software.

\textsuperscript{147} Bishop, \textit{supra} note 13, at 5-15.

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3. A “lopsided-margins test” to detect when the targeted party wins with unusually large margins

In summary, the success of a gerrymandering scheme depends on the ability of the redistricting party to create safe margins of victory for both parties, with larger margins for their opponents. This pattern of outcomes can be quantified by sorting the districts into two groups, by winning party. Each party’s winning vote shares can then be compared by what is said to be “the most widely used statistical test of all time”149: the t-test for comparing the averages of two groups of observations. In this way, the difference between each party’s winning margins will be used to carry out Test 2, which tests for intensive packing of one party’s voters.

4. The mean-median difference as a measure of skewness

Now that I have identified states in which Republicans or Democrats gained an asymmetric advantage, I can examine these states to test the validity of a simpler statistic that does not require computer simulation: the difference between the mean (i.e., average) and the median vote share150 for contested districts.151 The mean-median difference is a simple measure of asymmetry152 that allows for ready comparison with national standards. Notably, it does not require any inputs other than district-level election results for the state that is under examination.

As an example of the calculation, consider the 2012 Pennsylvania congressional election. The Democratic two-party share of the total vote in all eighteen districts was, in terms of percentages and sorted in ascending order:


150. The mean-median difference was originally suggested as a measure of partisan gerrymandering in Michael D. McDonald & Robin E. Best, Unfair Partisan Gerrymanders in Politics and Law: A Diagnostic Applied to Six Cases, 14 ELECTION L.J. 312, 312 (2015).

151. The presence of uncontested races reduces the value of the mean-minus-median statistic. In those cases, the partisan breakdown is not known with accuracy. Consider the example of a twenty-district state. Residents of an uncontested district would have voted at a rate of 80% for their party, instead of the nominal 100%. If their district were drawn differently, the appropriate mean for comparison would be based on the 80% figure and shift the overall mean by 1%.

152. The mean-median difference is a simple and old measure of “skewness,” a statistical term for asymmetry. See G. UDNY YULE & M.G. KENDALL, AN INTRODUCTION TO THE THEORY OF STATISTICS 162-63 (14th ed. 1968); David P. Doane & Lori E. Seward, Measuring Skewness: A Forgotten Statistic?, J. STAT. EDUC., July 2011, at 9-10; Karl Pearson, Contributions to the Mathematical Theory of Evolution—II: Skew Variation in Homogeneous Material, PHIL. TRANSACTIONS ROYAL SOC’Y, 1895, at 343, 374-76.
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34.4, 36.0, 36.6, 36.7, 38.3, 40.6, 41.5, 41.6, 42.8, 42.9, 43.2, 43.4, 45.2, 48.3, 60.3, 69.1, 76.9, 84.9, 90.5.

Races won by Republicans are indicated in italics and the two middle values are underlined. The median percentage is defined as the midpoint of the two middle values, 42.85%. The mean Democratic vote share is 50.5%. The difference between the median and the mean is 7.6%. This difference reflects the fact that counterintuitively, Republican vote shares were above average in considerably more than half of the districts: 72% (thirteen out of eighteen), to be exact.

The median serves as a measure of the overall behavior of the eighteen district-level elections. The goal of a gerrymander is to maximize the number of districts won, which occurs when the median outcome is more unfavorable to the opposing party than that party’s share of the vote. In other words, Pennsylvania’s Democratic voters were empowered as if they comprised 42.85% of voters, even though they actually comprised 50.5%. The difference, 7.6%, is the number of voters who were effectively disenfranchised. Since approximately 5.5 million Pennsylvanians cast votes in the 2012 congressional election, redistricting achieved an effect equivalent to over 400,000 Democratic voters casting their ballots for Republicans. The probability is less than 1% that this difference arose by chance.154

5. State-by-state comparisons of skewness with population clustering effects

To investigate the degree to which the mean-median difference arises as a function of population clustering patterns, I make comparisons between a variety of states and years. For the 2012 congressional elections, the nationwide mean-median difference was 4.3% favoring Republicans across all fifty states and 1.9% favoring Republicans in non-dys-proportional states. For Pennsylvania in 2012 the difference was 7.6%, greater than any of the other mean-median differences, and comparable to the other four dys-proportional states of Michigan (mean-median difference of 6.3%), North Carolina (7.3%), Ohio155 (6.3%), and Virginia (6.3%). Overall, these mean-median differences are three to four times those seen in non-dys-proportional states, indicating that within a single state, the effects of partisan gerrymandering can be three or

153. HAAS, supra note 30, at 53. Democratic two-party share is defined as the number of Democratic votes divided by the sum of Democratic and Republican votes, expressed as a percentage.

154. The level of statistical significance is calculated using Test 3 in Part III.A below, and Student’s t-distribution. For the original calculation of the t-distribution, see Student, The Probable Error of a Mean, 6 Biometrika 1, 1 (1908); see also LOWRY, supra note 149 (explaining Student’s t-distribution).

155. In Ohio, one race (the eleventh district) was uncontested and won by a Democrat, Marcia Fudge. HAAS, supra note 30, at 48.
four times larger than the effects of population clustering. Indeed, as stated previously, redistricting in a handful of states can generate a greater deviation from symmetry than population clustering in all fifty states combined.

III. Three Quantitative Tests for the Detection of the Effects of Partisan Gerrymandering

I use the two Analyses to propose three tests. The three tests have several advantages. First and foremost, they are simple to apply. None of the three tests requires the detailed drawing of maps. Because the tests can be stated with mathematical exactness, they can also provide a manageable standard for gerrymandering cases, yielding predictable and sensible results—and unambiguous guidance to legislatures and judges. The tests are based on election outcomes and therefore can be employed separately from, or in conjunction with, geographic and other criteria. An online calculator for these tests is available at http://gerrymander.princeton.edu.

A. Converting the Analyses to Practical Tests

I use the Analysis of Effects, which is based on numerical simulation of seat outcomes, to construct Test 1, the excess seats test. I use the Analysis of Intents, which identifies narrow-but-reliable wins as a hallmark of gerrymandering, to construct two tests: Test 2, the lopsided outcomes test; and Test 3, a reliable-wins test.

Test 1 (the excess seats test): Calculate whether the outcome of an election after redistricting was dys-proportional relative to a simulated seats/votes curve and whether that outcome favors the redistricting party. For a state containing $N$ districts, calculate the difference between the actual seats and the simulated expected number and divide by the standard deviation to obtain the difference, $\Delta$.

Test 2 (the lopsided outcomes test): Compare the difference between the share of Democratic votes in the districts that Democrats win, and the share of Republican votes in the districts that Republicans win. This test works because in a partisan gerrymander, the targeted party wins lopsided victories in a small number of districts, while the gerrymandering party’s wins are engineered to be relatively narrow. To compare the winning vote shares for the two parties, I use a grouped $t$-test, an extremely common statistical test.

Test 3 (the reliable-wins test): Systematic rigging of total statewide outcomes occurs by the construction of districts that offer secure wins for the party in

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156. This $\sigma_1$ can be calculated according to the formula for sigma described above, see supra note 107 and accompanying text, or by numerical simulation, see supra Part IIA.3.

157. See supra Part IIB.

158. See LOWRY, supra note 149.
control of the map. These wins would be wide enough to guarantee victory, but not so wide as to waste votes that could be used to shore up other districts. How this intent is detected depends on whether the state’s partisan vote is closely divided, or whether one party is dominant. In a closely divided state, reliable wins occur when the average and median vote differ from one another. In a state that is dominated by one party, reliable wins occur when that party’s strength is spread highly evenly across districts.

- **In a closely divided state**: Calculate the difference between a party’s statewide average (i.e., mean) district vote share on the one hand, and the median vote share it receives on the other. In this situation, a systematic gerrymander can be detected when a party’s median vote share is substantially below its average vote share across districts. For this test, calculate Delta by dividing the mean-median difference by $\sigma_3$, which is defined as $0.756 \times (\text{standard deviation of vote share across all } N \text{ congressional districts in a state})/\sqrt{N}$.  

- **In a state where the redistricting party is dominant**: Calculate the standard deviation of the redistricting party’s vote share in the districts that it wins. Calculate the standard deviation of the party’s vote share in the districts that it wins nationwide. Compare these two standard deviations using a well-established testing tool, the chi-square test for comparison of variances, to define zones of chance.

Test 1 evaluates whether a party gained a significant advantage in terms of seats and calculates the size of the effect. Tests 2 and 3 determine whether the pattern of data could have arisen by chance; if not, this indicates an intent to gerrymander. A residual possibility exists of a false-positive result, i.e., identifying that a gerrymandering event occurred when in fact it did not. To reduce the possibility of such a false alarm, partisan gerrymandering could be assessed by evaluating both Test 2 and Test 3. If Delta is set to standard levels of statistical significance in 2012, six states met both the Test 2 and Test 3

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159. This is the mean-median test described by the Author in Wang, supra note 29, and in McDonald & Best, supra note 150, at 321-29.


161. See GEORGE W. SNEDECOR & WILLIAM G. COCHRAN, STATISTICAL METHODS 251-52 (8th ed. 1989); Karl Pearson, On the Criterion That a Given System of Deviations from the Probable in the Case of a Correlated System of Variables Is Such That It Can Be Reasonably Supposed to Have Arisen from Random Sampling, 5 Phil. Mag. Series 157, 163-67 (1900) (describing the original mathematical derivation of the chi-square statistic, the practical use of which, for purposes of analyzing redistricting, is better accomplished using the online calculator at Sam Wang, Gerrymandering Demo, PRINCETON UNIV., http://gerrymander.princeton.edu (last visited June 6, 2016)).

162. A typical level of statistical significance is to set the threshold for Delta so that chance would give the observed result 5% of the time or less. Whether this occurs depends on Delta, which increases in proportion to the square root of the number of districts. See
B. Advantages and Disadvantages of the Three Tests

The tests proposed here all have several advantages. First, the tests do not require the detailed drawing of maps. Second, because they are derived from election results only, the tests can be applied independently of evaluation of intent. Third, because their results are highly correlated, in situations where one test is unsuitable, another can be used instead. In this way the tests can be used separately or combined to reduce the risk of falsely identifying a gerrymander where none occurred, or conversely, failing to detect a gerrymander where one did occur. Finally, because the three tests do not use geography, they can easily be combined with other standards which may require circuitous geographic boundaries, such as state-mandated requirements, section 2 of the Voting Rights Act, and other precedents that exist in federal law.

In choosing which test to apply, the judge (or other evaluator of a districting plan) should take the following advantages and disadvantages into account.

Test 1’s most powerful use is to obtain an exact range for the appropriate number of seats for a given vote share. It addresses whether a redistricting scheme leads to an elected delegation that deviates from national districting norms; i.e., it measures effects. Test 1 can always be calculated from a set of election returns. Because it uses data from other states, it has the advantage of taking into account the overall nationwide demographic character of districts. Therefore, it has the virtue of measuring effects that are in addition to those that arise naturally from population clustering. However, because it requires computer simulation, it requires the use of a computer program (which is freely accessible at gerrymander.princeton.edu and can also be obtained by contacting the Author).

Test 2 has the advantage of simplicity: it can be worked out using a spreadsheet program such as Microsoft Excel that can perform a two-sample test. Delta is evaluated by comparison with significance values for the t-distribution. For Tests 2 and 3, statistical significance is typically reached when Delta exceeds 1.75.

163. The three tests proposed here address the overall apportionment plan, but do not cover the case of individual self-dealing in single districts. Local laws may provide additional constraints. For example, the current congressional districts in Florida do not violate my three tests. Nonetheless, the Florida Supreme Court has held that the map violates the Florida Constitution redistricting provisions. See League of Women Voters v. Detzner, 172 So. 3d 363, 427 (Fla. 2015) (relying on Fla. Const. art. III, § 20(a) which mandates that “[n]o apportionment plan or individual district shall be drawn with the intent to favor or disfavor a political party or an incumbent”). This stricter standard extends a mandate for competitive races to the level of single districts.
Apart from the t-test. If such a program is not available, it can be done using a hand calculator and a table of statistical values. It directly tests for noncompetitive races, a mainstay of gerrymandering. It identifies partisan asymmetry, though not bipartisan gerrymanders in which individual candidates of both parties benefit. Test 2 has the disadvantage that it can only be used if both parties win at least 2 seats each, since this is required to calculate standard deviations, a necessary step of the test.

Test 3 measures the reliability of wins for the redistricting party. Like Test 2, it is simple to calculate. Test 3 can always be done, since it is calculated using a state’s district-level results. In the case of the mean-median difference, it does not rely on any data from other states and is therefore self-contained. In the case of the chi-square test, national data must be used to provide a standard for comparison.
### Table 2

Results of Three Tests for Partisan Asymmetry for the Congressional Elections of 2012

<table>
<thead>
<tr>
<th></th>
<th>Test 1 (Simulation)</th>
<th>Test 2 (Lopsided Margins)</th>
<th>Directly From Election Returns</th>
<th>Imputing Uncontested Races</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total seats</td>
<td>Simulated Average</td>
<td>Δ (Difference Divided by Sigma)</td>
<td>Δ (Difference Divided by Sigma)</td>
</tr>
<tr>
<td>Arizona</td>
<td>9</td>
<td>2.96</td>
<td>D by 2.7</td>
<td>63.1% 66.6% 9.5% D by 0.4</td>
</tr>
<tr>
<td>Florida</td>
<td>27</td>
<td>11.73</td>
<td>R by 1.3</td>
<td>73.0% 67.4% 7.4% R by 0.8</td>
</tr>
<tr>
<td>Illinois</td>
<td>18</td>
<td>10.04</td>
<td>D by 1.8</td>
<td>66.2% 62.1% 4.9% R by 0.8</td>
</tr>
<tr>
<td>Indiana</td>
<td>9</td>
<td>3.02</td>
<td>R by 1.3</td>
<td>65.1% 59.5% 3.1% R by 1.8</td>
</tr>
<tr>
<td>Maryland</td>
<td>8</td>
<td>6.11</td>
<td>D by 1.2</td>
<td>70.4% 66.5% 3.9% R by 1.8</td>
</tr>
<tr>
<td>Michigan</td>
<td>14</td>
<td>6.97</td>
<td>R by 2.0</td>
<td>74.4% 58.9% 4.9% R by 1.2</td>
</tr>
<tr>
<td>North Carolina</td>
<td>13</td>
<td>5.94</td>
<td>R by 2.1</td>
<td>70.2% 57.3% 6.9% R by 1.9</td>
</tr>
<tr>
<td>Ohio</td>
<td>16</td>
<td>6.48</td>
<td>R by 2.4</td>
<td>80.2% 62.2% 7.5% R by 2.4</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>18</td>
<td>8.14</td>
<td>R by 2.9</td>
<td>76.3% 59.5% 5.5% R by 3.1</td>
</tr>
<tr>
<td>Texas</td>
<td>36</td>
<td>8.68</td>
<td>D by 2.3</td>
<td>71.4% 72.1% 4.5% D by 0.2</td>
</tr>
<tr>
<td>Virginia</td>
<td>11</td>
<td>4.56</td>
<td>R by 1.8</td>
<td>70.9% 58.8% 5.6% R by 2.1</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>8</td>
<td>3.64</td>
<td>R by 0.9</td>
<td>68.9% 59.6% 3.8% R by 2.4</td>
</tr>
</tbody>
</table>

In all cases, the last column gives the difference between expectations and actual results, expressed in units of sigma, the standard deviation, to give a measure that is comparable across the three tests. Test 3 starts from raw percentage results and the last column assumes voters in uncontested races are distributed 75%-25% for the winning party. The boldface underlined entries indicate statistically significant results. Test 2 could not be done for Maryland because the grouped t-test requires each group to include at least two wins.
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C. Three Examples: The Original Gerry-mander, Arizona State Legislative Districts, and Maryland Congressional Districts

To examine the general applicability of these tests, let us consider three examples: (1) the original Gerry-mander of 1812, (2) the post-2010 Arizona state legislative districts that was considered by the Supreme Court in the 2015 Term,164 and (3) the post-2010 Maryland congressional districts, which the Supreme Court recently remanded for consideration by a three-judge court.165

Example 1: The original “Gerry-mander,” the Massachusetts State Senate Election of 1812. Test 1 is evaluated by starting from the fact that there were 18 races.166 The average expectation of a nearly evenly divided popular vote is 9 seats for each party. The upper theoretical value to sigma is $0.5\sqrt{18} = 2.1$ seats; computational simulation reveals a true value of sigma of 1.4 seats. The Federalists won only 5 seats,167 and therefore Test 1 is met to a standard of $(9-5)/1.4 = 2.9$ sigma, statistically significant.

For Test 2, the Federalists won five races (which accounted for eleven districts); in these races, their two-party vote share averaged 55.6%, with a standard deviation of 4.6%. The Democratic-Republicans won thirteen races (which accounted for twenty-nine districts), with an average vote share of 70.7% and a standard deviation of 5.3%. The resulting Delta (also called t-score) is 5.5, and therefore Test 2 is met to a standard of 5.5 sigma. This is an unusually high level of significance; this result is reached by chance 0.0025% of the time.

Test 3 should not be used because districts are not equal in size. In 1812, the number of votes per legislator ranged from Dukes/Nantucket (1078 votes cast in total for one legislator) to Franklin (4469 votes for one legislator).168

Example 2: Arizona State Legislative Districts. After the 2010 census, the Arizona Independent Redistricting Commission, which is composed of members of both major political parties, drew House and state legislative districts.169 A case recently decided before the Supreme Court, Harris v. Arizona

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166. See Lampi Collection, supra note 4 (listing election results). For the calculation of Test 1, each district election is used as one data value. For the number of districts, see Griffith, supra note 1, at 62.
167. Lampi Collection, supra note 4.
168. Id. In that election, multimember districts of unequal population were allowed. Equipopulation districts were not required until the Supreme Court held that malapportionment claims were justiciable in Baker v. Carr, 369 U.S. 186 (1962), and later developed the one person, one vote standard in Reynolds v. Sims, 377 U. S. 533 (1964). For a history of the one person, one vote standard, see Samuel Issacharoff et al., The Law of Democracy: Legal Structure of the Political Process 126-213 (4th ed. 2012).
Independent Redistricting Commission, concerned whether “the desire to gain partisan advantage for one political party justifies creating legislative districts of unequal population that deviate from the one-person, one-vote principle of the Equal Protection Clause.”\textsuperscript{170} In that case, plaintiffs contended, and the District Court of Arizona “assume[d] without deciding[,] that partisanship is not a legitimate reason to deviate from population equality.”\textsuperscript{171} The Independent Redistricting Commission, by contrast, contended that it constructed districts of unequal population to comply with section 5 of the Voting Rights Act.\textsuperscript{172} In a unanimous opinion, the Court held for the Commission, allowing the district map to stand.\textsuperscript{173}

Although the issue at hand was the creation of overpopulated districts, neither side contested in federal courts the premise that the Commission created a partisan advantage. This turned out to be a key point, since the Court noted that “[e]ven assuming, without deciding, that partisanship is an illegitimate redistricting factor, appellants have not carried their burden.”\textsuperscript{174} The question bears examination: Did redistricting actually create a partisan advantage in the first place? This question can be tested by examining state senate races, of which there is one for each of Arizona’s thirty legislative districts; or state House races, which elected two Representatives for each of the same thirty districts.

Test 1 relies on computer simulation using other comparable districts as a source of hypothetical districts. The statewide two-party popular vote totaled 56.3\% for Republicans and 43.7\% for Democrats, yielding seventeen seats for Republicans and thirteen seats for Democrats. Because other states have different districting systems (for instance with different numbers of people per district), data is not available to allow simulation of the seats/votes relationship. However, a simpler calculation is possible: proportional representation would predict 16.9 seats for Republicans. Therefore, the election result is almost perfectly eu-proportional, and therefore does not require further analysis.

For Test 2, appellants asserted that the Democratic Party benefited from the redistricting.\textsuperscript{175} In Arizona’s state senate races in 2014, the average winning Republican vote share was 73\%, while the average winning Democratic vote

\textsuperscript{172} Id. at 1046.
\textsuperscript{173} Harris, 136 S. Ct. at 1305.
\textsuperscript{174} Id. at 1310.
\textsuperscript{175} Brief for Appellants, supra note 170, at 17.
share was 72%. This difference—one percentage point—is not statistically significant. In state House races, Republicans won with an average of 66% in those districts they won, while Democrats won with 64% in those districts they won; again, the difference was not statistically significant.

For Test 3, the mean Democratic vote share across thirty districts was 50.1%. Therefore, the state’s votes were closely divided, and the appropriate test is the mean-median difference. The median Democratic vote share was 45.6%, for a mean-median difference of 3.3% (4.1% with imputation of uncontested races) in a direction that favors Republicans. This difference works against Democrats and therefore is in the wrong direction.

Based on the foregoing, Arizona senate districts fail all three tests. Therefore, the contention that Democrats benefited in a dys-proportional manner is not supported, and the Supreme Court was correct in pointing out the absence of undue partisanship. If the Commission was trying to engineer a map that systematically disfavored Arizona Republicans, it did a poor job.

Example 3: Maryland Congressional Districts. Maryland has eight congressional districts. Steven Shapiro and other plaintiffs filed suit in district court that the post-2010 districting plan violated their rights to political association under the First Amendment. The district court dismissed the complaint, and the U.S. Court of Appeals for the Fourth Circuit affirmed its dismissal. However, in December 2015 the Supreme Court reversed the decision, remanding the case to a three-judge court for further consideration.

In Maryland, Democrats typically win around 60% of the vote at a statewide level—the same as the margin needed for a safe victory. Artful arrangement is accomplished and can be detected in the form of many districts of near-identical partisan composition (Figure 5).

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179. Shapiro, 136 S. Ct. at 456.
Test 1 identifies Maryland as a gerrymander. In the pre-redistricting election of 2010, Democrats won 63.2% of the statewide vote and six seats, compared with a simulated average of 6.1 seats—not statistically significant (Table 1). However, after redistricting, in 2012 Democrats won 65.5% of the statewide vote and won seven seats, compared with a simulated average of 6.1 seats. The value of Delta was 1.2 favoring Democrats, not quite statistically significant. In 2014, Democrats’ vote share declined to 58.1%, but they retained all seven of their seats. In this case, the simulated average was 5.1 seats, and the value of Delta was 2.4, statistically significant. These results indicate that redistricting gained Democrats a one-seat advantage in 2012, a strong Democratic year, and that this advantage was retained in the national wave election of 2014 that swept dozens of Republicans into office in states outside Maryland.

Test 2 cannot be applied because the standard deviation of the Republican winning vote share cannot be calculated with only one Republican congressman; at least two values are required to calculate a standard deviation.

Test 3 should be done for the case of partisan dominance, a situation that calls for the chi-square test to test whether Democratic votes are spread

unusually uniformly across congressional districts. Figure 6 shows the standard deviation. The standard deviation of Maryland Democrats' winning vote share in seven districts was 6.6% in 2012 and 7.3% in 2014. I compared the variability of Maryland Democratic districts with the variability of Democratic districts nationwide. The values for Maryland fall outside the zone of chance.

Figure 6
Standard Deviation of Democratic Vote Share over Time

The black jagged line at top indicates the standard deviation of the Democratic vote share nationally. Black circles indicate the standard deviation for Maryland districts. The gray shaded area indicates the zone of chance for Test 3, which is bounded below by a second black jagged line. The 2012 and 2014 elections fall outside the zone of chance and furthermore pass Test 1 for significance.

Maryland's standard deviations would have arisen by chance in only 2.8% of cases in 2012, and 1.7% of cases in 2014. A third year, 2004, also showed an

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183. The standard deviation is the square root of the variance.
184. For a lower one-tailed test at significance level p<0.05, the lower bound of the zone of chance is equal to (national standard deviation)*\sqrt{(2.167/(N - 1))}. See Chi-Square Distribution Table (n.d.), http://sites.stat.psu.edu/~mga/401/tables/Chi-square-table.pdf (giving a table of critical values above which the chi-square score is...
unusually low standard deviation. These findings show that the Democrats’ partisan advantage was achieved by spreading their partisan support in a highly even manner across their winning districts.

IV. Discussion

In this Article, I have presented three tests for rapid identification of partisan gerrymanders. My tests can be used to evaluate intents and effects; the two prongs articulated in Davis v. Bandemer. The two intents tests can be done with computing resources already available on a judge’s or clerk’s desk, and the effects test requires some additional software. All three tests rely on well-established statistical principles. The tests measure different aspects of partisan asymmetry and therefore fall within the scope of principles that have been expressed by the Supreme Court. I suggest that these tests may constitute a manageable standard for courts to evaluate the impact of a state’s districting scheme on its residents’ equal protection and First Amendment rights.

The broader implications of this Article are threefold. First, I have used statistical science to express the idea that a pattern of election results might have arisen by chance, and therefore not warrant judicial intervention. By establishing “zones of chance” in which the partisan impacts of a districting plan are ambiguous, my tests can help a judge evaluate whether an identifiable injury has occurred in the first place. Second, my statistical analysis shows that in 2012, the effects of partisan asymmetry were so large as to exceed the effects of population clustering across the whole nation. This demonstrates the importance of measuring the degree of distortion from the natural relationship between votes and representation. Third, an intents-and-effects standard based on my tests is unambiguous and may mitigate the need to demonstrate predominant partisan intent. For these reasons, my statistical tests comprise a

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185. Without partisan intent, the Maryland standard deviation would still be expected to fall outside the zone of chance in five percent of cases—one in twenty. Maryland’s 2004 congressional delegation was within the zone of chance by Test 1, indicating that the result of Test 3 is a chance result, i.e., a "false positive." A version of this software is available on Github at Wang, supra note 110, and a web browser-based version is available at Wang, supra note 161.
valuable and timely addition to the judge's toolkit for rapid and rigorous identification of partisan gerrymanders.

A. Allowing for Ambiguity

My statistical analysis of the effects of gerrymandering may be of particular relevance to First Amendment analysis, which "allows a pragmatic or functional assessment that accords some latitude to the States." By allowing for a normal amount of statistical variation, the three tests proposed in this Article build in zones of chance where any of a range of outcomes would lead to an acceptable amount of asymmetry.

Any statistical approach contains some possibility of accidentally identifying gerrymandering where it does not exist ("false positives"), or missing cases where it did occur ("false negatives"). Tests may also sometimes not be usable, such as Test 2, when one party only wins one seat. For these reasons, I provide two separate tests of intents. These tests are oriented toward the outcomes of elections rather than the specifics of map boundaries or district procedures. The tests hew closely to the electoral goals of redistricters and do not rely on geographically oriented approaches which require normative assumptions of what constitutes good districting procedure.

B. What Is the Role of Intent?

Over time, the Court's decisions have set a standard for intent so stringent that it cannot be satisfied. The resulting high bar to proving injury requires more than simply showing that partisanship was one of multiple factors and is a far higher bar than the evaluation of disparate impact alone. Such a demanding standard may have been appropriate in the absence of legislative guidance or a large body of court precedent. In the Bandemer and Vieth framework, the lack of simple and reliable tests made it necessary to assess the link between redistricters' actions and the injury. Indeed, current approaches to proving gerrymanders focus on intent, are diverse in approach, and sometimes do not agree with one another. In contrast, a statistically based test may provide a more satisfactory route to satisfying the intent standard.

188. See Davis v. Bandemer, 478 U.S. 109, 129 (1986); Vieth, 541 U.S. at 284-86 (plurality opinion) (reviewing the difficulty of meeting a standard of "predominant intent").
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The facts in LULAC exemplify ambiguous intent.\(^{190}\) For example, the Republican majority “honored” some requests by individual Democratic legislators in the districting process.\(^{191}\) However, partisan gerrymandering is, by its nature, prone to satisfying such dual interests because a party as a whole has motivations that can align with those of selected individual legislators of the opposing party.\(^{192}\) Therefore intent is most fairly evaluated at the state level or at the individual level, but not both at the same time. In addition, the majority in *Crawford v. Marion County Election Board* held that partisan intent alone is an insufficient reason to strike down voting restrictions.\(^{193}\)

Quantitative measures of intent, such as my proposed Tests 2 and 3, allow the identification of patterns of districting that are highly unlikely to have arisen by chance, thereby providing concrete evidence that a legislature or other district-drawing body acted specifically to produce partisan outcomes. Satisfaction of such a rigorous standard should open the way to examining other facts as additional evidence for probable (not predominant) intent.

Furthermore, I suggest that districting can impose a burden on a group’s representational rights whether or not the offense was intentional. Even where intentions are nonpartisan, bipartisan, or unknown, the effect of a districting plan with partisan asymmetry is to produce legislative blocs whose size is unrepresentative of the popular will. The construction of a reliable measure of effect provides clear guidance when an injury has taken place and a template for how the injury can be repaired. Just as a road worker may act to right an upended orange traffic cone even if she does not know how the cone came to be tipped over, a court may act when effects are sufficiently strong, as in disparate impact cases in racial discrimination cases.\(^{194}\) Although partisan gerrymandering cases are governed by different doctrine (constitutional) than

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190. League of United Latin Am. Citizens (LULAC) v. Perry, 548 U.S. 399, 417-18 (2006) (citing the honoring of Democratic state legislators’ requests as indicating that partisan gain was not the redistricters’ sole motivation).

191. Id. at 418.

192. For a discussion of mixed partisan motivations, see note 145 above and accompanying text.


194. In one recent example of a racial discrimination case, the Supreme Court held that demonstrating disparate impact was sufficient to prove discrimination, and that it was only necessary to demonstrate that the effect arose as a consequence of actions, as opposed to explicit racial intent. See Tex. Dep’t of Housing & Comm. Aff. v. Inclusive Comms. Project, Inc., 135 S. Ct. 2507, 2517-19 (2015). The Court held that in light of the results-oriented statutory language in the Fair Housing Act, a showing of disparate impact was sufficient to warrant a remedy, even without discriminatory intent. Id. I argue that if gerrymandering has a sufficiently large effect on a party’s supporters, such an injury should still be remedied even when redistricters are not motivated purely by partisan intent.
C. Evaluating the Partisan Impact of District Maps Before Implementation

Although in this Article I use election results to calculate the three tests, the tests could alternatively use other inputs. For example, to rule out the possibility that the tests may be influenced by variations in the quality of specific candidates, it would be possible to use district-level presidential vote shares as inputs.195

In current federal precedent, the need for redrawing a set of districts often relies on elections that have already taken place. However, by that time, an injury to voters has already occurred. To preempt such an injury from happening in the first place, the three tests could be calculated using information that is available before an election. Under the First Amendment rationale of not penalizing groups for their partisan preference, party registration might be used as an input to calculate the three tests. Political scientists, redistricters, and commercial redistricting software also use other variables to predict overall partisan preference; these predictions could also serve as inputs to the tests. Doing so would allow a hypothetical districting scheme to be assessed before it has passed into law.

The standards presented here can quantify the benefits of reform efforts directed at reducing the likelihood of partisan gerrymandering. One such route is the establishment of nonpartisan districting commissions that remove districting from the direct control of legislators. In 2008, California voters approved Proposition 11, which established the California Citizens Redistricting Commission.196 The commission is composed of fourteen members who are drawn from members of the general public, including five Democrats, five Republicans, and four members who decline to state a partisan loyalty.197 The commission’s mandate is to draw districts that respect principles of contiguity, compactness, and representation of a community’s interests.198 The resulting congressional districts have become more

195. Justice Kennedy, joined by Justices Souter and Ginsburg, explained that with respect to a partisan gerrymandering claim, “such a challenge could be litigated if and when the feared inequity arose.” LULAC, 548 U.S. at 420. Redistricting software is capable of using quantities such as the presidential vote share to estimate the partisan tendency of a hypothetical district. Redistricters use such measures to judge the likely outcome of a district and could use them as inputs to my three tests to evaluate a districting plan before it is implemented.


197. CAL. CONST. art. XXI, § 2(c)(2).

198. Id. § 2(d).
competitive: margins of victory have become smaller, and incumbents have lost their reelection races at higher rates than before the formation of the commission.199 Like the Arizona commission, the work of the California commission has led to closer races and more eu-proportional overall outcomes than precommission maps.

These tests could also be used in approaches that leave districting under the control of state legislators, but place constraints on how and what they produce. Such an approach has been taken in Florida, where ballot initiatives known as Amendments 5 and 6 were passed in 2010 and precleared by the Department of Justice a year later, becoming Article III, sections 20 and 21 of the Florida Constitution.200 Together with Article III, section 16,201 the Florida Constitution stipulates that district lines must be contiguous, compact, and use existing political geographical boundaries where available.202 Districts also may not be drawn to "favor or disfavor a political party or incumbent."203 The resulting plans are subject to review by the Florida Supreme Court, leading either to approval or return to the legislature for a further attempt to meet districting criteria.204 The tests described in this Article could be useful in identifying statewide partisan favor. Individual districts would still need to be evaluated separately, for example to comply with Voting Rights Act restrictions and other principles set down in federal or state law. My tests, which address the properties of combinations of districts, can complement these other constraints without conflict.

**Conclusion**

Partisan gerrymandering distorts relationships between voting and representation that would otherwise arise naturally, generates seats that are unresponsive to shifts in public opinion, and chills the freedom of voters to associate with a political party of their choosing. The health of democratic processes would be considerably improved by reducing the ability of legislative

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201. Fla. Const. art. III, § 16.

202. Id. §§ 20-21.

203. Id. §§ 20(a), 21(a).

204. Id. §§ 16(c), 16(d).
processes to impose partisan distortions on redistricting maps. My three tests for asymmetry may contribute to a manageable standard for identifying partisan gerrymanders, with the eventual goal of reducing or eliminating them.